Quantum mechanics expressed in terms of the approach "Emission & Regeneration" UFT.

Osvaldo Domann

odomann@yahoo.com

First publication 2003 and last revision June 2022

(This paper is an extract of [11] listed in section Bibliography.)

Abstract

Quantum mechanics differential equations are based on the de Broglie postulate. This paper presents the repercussions on quantum mechanics differential equations when the de Broglie wavelength is replaced by a relation between the radius and the energy of a particle. This relation results from the theoretical work [11] about the interaction of charged particles, where the particles are modelled as focal points of rays of fundamental particles with longitudinal and transversal angular momentum. Interaction of subatomic particles is described as the interaction of the angular momenta of their fundamental particles. Based on the finding that electrons and positrons neither attract nor repel each other for the distance between them tending to zero, and that protons are swarms of electrons and positrons, the energy levels of the orbital electrons of the hydrogen are explained with the number of positrons of the proton that interact with the orbital electron. All four known forces are the result of electromagnetic interactions, so that only QED is required to describe them. The potential well of an atomic nucleus is shown with the regions that are responsible for the four type of interactions defined in quantum mechanics. Also the compatibility of the gravitation model derived in [11] with quantum mechanics is shown, model where gravitation is the result of the reintegration of migrated electrons and positrons to their nuclei.

Contents

1 Introduction.

Quantum mechanics differential equations are based on the de Broglie postulate. In the theoretical work [3] about the interaction of charged particles, where particles are represented by a non local model emitting and absorbing continuously fundamental particles, a relation between the radius r_o and the energy of a particle is derived.

$$r_o = \frac{\hbar\, c}{E} \qquad with \qquad E = \sqrt{E_o^2 + E_p^2} \quad the\ relativistic\ energy. \tag{1}$$

This relation is used instead of the de Broglie wavelength, to build wave packages with a Gauss distribution, and to derive the corresponding probability differential equations of quantum mechanics.

The effects on the uncertainty relations and the most important quantum mechanics operators are presented.

Note: When deriving the wave-package with the radius-energy relation, the mass of a particle is considered as concentrated in a sphere with a diameter equal approximately to two times the radius r_o given by the radius energy-relation. This is not according to the approach that represents particles as Focal Points which led to the radius-energy relation where the mass (energy) of a particle is distributed from r_o to infinity, outside the sphere with radius r_o.

1.1 General considerations.

To make use of the of Fourier-Transformation, the movement of a particle is first described as a sequence of particles represented by a sinus wave, having a wavelength λ equal to $2\pi r_o$. Then the Fourier-Transformation of a wave package of sinus waves with a Gauss shaped amplitude is build.

We have that

$$\lambda = 2\pi r_o = 2\pi \frac{\hbar\, c}{E_{rel}} \qquad with \qquad E_{rel} = \sqrt{E_o^2 + E_p^2} \tag{2}$$

with

$$E_o = m_o\, c^2 \qquad E_p = p\, c \qquad p = \frac{m_o\, v}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

The sinus wave on the x-axis is

$$\xi_x = A\, e^{i(k_x\ x - \omega_x\ t)} \qquad with \qquad k_x = \frac{2\pi}{\lambda_x} \qquad and \qquad \omega_x = 2\pi\ \frac{v_x}{\lambda_x} \tag{4}$$

If we now introduce in the expression that $\lambda_x = 2\pi r_{o_x} = 2\pi\hbar c/E_{rel_x}$ we get

$$\xi_x = A\ exp\left[i\frac{c}{\hbar}\left(\frac{E_{rel_x}}{c^2}\ x - \ \frac{v_x}{c^2}\ E_{rel_x}\ t\right)\right] \tag{5}$$

or

$$\xi_x = A\ exp\left[i\frac{c}{\hbar}\left(\frac{E_{rel_x}}{c^2}\ x - \ p_x\ t\right)\right] \tag{6}$$

with

$$E_{rel_x} = m_o\ c^2\left(1 - \frac{v_x^2}{c^2}\right)^{-1/2} \qquad and \qquad p_x = \frac{v_x}{c^2}\ E_{rel_x} \tag{7}$$

with E_{rel_x} the relativistic energy of the particle on the x-axis.

Note: The wave-length used by Schroedinger is based exclusively on the kinetic energy E_{kin_x} for the non-relativistic case as follows.

$$\lambda = 2\pi r_o = 2\pi\frac{\hbar\ c}{E_{rel}} \quad with \quad E_o = 0 \quad and \quad E_p = p\ c \quad where \quad p = m\ v \tag{8}$$

The proposed approach includes for the calculation of the wave-length the total energy with the rest energy of a particle. For the relativistic cases we get

$$\lambda = 2\pi r_o = 2\pi\frac{\hbar\ c}{E_{rel}} = 2\pi\frac{\hbar}{m\ c\ \gamma} \quad with \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{9}$$

For $v \to c$ we get that $\lambda \to 0$.

1.2 The wave package.

We define the Fourier-Transformation of a wave package [1,2]; on the x-axis as

$$\phi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\kappa_x(p_x)\ exp\left\{i\frac{c}{\hbar}\left[m_{rel_x}(p_x)\ x - \ p_x\ t\right]\right\}dp_x \tag{10}$$

with a Gauss distribution $\kappa_x(p_x)$ on the p_x-axis

$$\kappa_x(p_x) = B\ exp\ \left\{-\frac{(p_x - p_{x_o})^2}{4(\Delta p_x)^2}\right\} \tag{11}$$

and the dispersion $m_{rel_x} = m_{rel_x}(p_x)$ with

$$m_{rel_x} = \frac{E_{rel_x}}{c^2} \qquad m_{rel_x} = m_{rel_x}(p_x) = \frac{1}{c^2}\sqrt{E_o^2 + p_x^2c^2} \qquad and \qquad E_o = m_oc^2 \tag{12}$$

Because of symmetry reasons we can write also a wave package

$$\psi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \chi_x(m_{rel_x})\ exp\left\{i\frac{c}{\hbar}\left[m_{rel_x}\ x - \ p_x(m_{rel_x})\ t\right]\right\} dm_{rel_x} \tag{13}$$

with the Gauss distribution on the m_{rel_x}-axis

$$\chi_x(m_{rel_x}) = A\ exp\ \left\{-\frac{(m_{rel_x} - m_{rel_{x\,o}})^2}{4(\Delta m_{rel_x})^2}\right\} \tag{14}$$

and the dispersion

$$p_x(m_{rel_x}) = c\ \sqrt{m_{rel_x}^2\ -\ m_o^2} \qquad and \qquad m_o = \frac{E_o}{c^2} \tag{15}$$

2 Differential equations.

2.1 Unrestricted differential equations.

In this and the following section the probability differential equations are derived. The differential equations are classified into unrestricted and non-relativistic. Then they are subclassified in groups of general, time or space independent.

The unrestricted differential equations are valid for the whole range of speed $0 \leq v \leq c$.

We start with the wave package

$$\psi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \chi_x(m_{rel_x})\ exp\left\{i\frac{c}{\hbar}\left[m_{rel_x}\ x - \ p_x(m_{rel_x})\ t\right]\right\} dm_{rel_x} \tag{16}$$

with

$$m_{rel_x} = \frac{E_{rel_x}}{c^2} \qquad and \qquad p_x(m_{rel_x}) = c\ \sqrt{m_{rel_x}^2 - m_o^2} \tag{17}$$

with

$$E_{rel_x} = E_o + E_{kin_x} = \sqrt{E_o^2 + E_{p_x}^2} \qquad E_o = m_o c^2 \qquad E_{p_x} = p_x\ c \tag{18}$$

For the unrestricted range of velocities $0 \leq v \leq c$ we have that

$$p_x = \frac{v_x}{c^2} E_{rel_x} \tag{19}$$

and E_{kin_x} represents the kinetic energy for the whole range of speed.

2.1.1 The wave equation.

The wave differential equation we obtain by derivation of ψ_x two times versus t and two times versus x. The results are then connected through

$$p_x = \frac{v_x}{c^2} E_{rel_x} \tag{20}$$

We get

$$\frac{\partial^2}{\partial x^2}\psi_x \;=\; \frac{1}{v_x^2}\,\frac{\partial^2}{\partial t^2}\psi_x \tag{21}$$

For $v_x \to c$ we have

$$\frac{\partial^2}{\partial x^2}\psi_x(x,t) \;=\; \frac{1}{c^2}\,\frac{\partial^2}{\partial t^2}\psi_x(x,t) \tag{22}$$

the well known wave equation

2.1.2 The time independent differential equation.

Time independent differential equations are deduced deriving one time and two times the wave function ψ_x.

a) We derive the wave function ψ_x one time versus x and get the following time independent differential equation on the x coordinate

$$\frac{\partial}{\partial x}\psi_x = \frac{i}{\hbar\, c}\; E_{rel_x}\; \psi_x = \frac{i}{\hbar\, c}\; (E_o + E_{kin_x})\; \psi_x \tag{23}$$

E_{kin_x} represents the kinetic energy for the whole range of speed, relativistic and non-relativistic.

The equation writes for conserved systems with the potential energy $U(x)$ as

$$-\,i\,\hbar\,c\frac{\partial}{\partial x}\psi_x - E_o\;\psi_x \;+\; U(x)\;\psi_x = [E_{kin_x} \;+\; U(x)]\;\psi_x = E_{tot}\;\psi_x \tag{24}$$

where E_{tot} is the conserved energy.

b) We derivate the wave function ψ_x two times versus x and get the following time independent differential equation on the x coordinate

$$\frac{\partial^2}{\partial x^2}\psi_x = -\;\frac{c^2}{\hbar^2}\; m_{rel_x}^2\;\psi_x \tag{25}$$

With

$$m_{rel_x} = \frac{1}{c^2}\sqrt{E_o^2 + E_{p_x}^2} \qquad\qquad E_o = m_o c^2 \qquad and \qquad E_{p_x} = p_x\, c \tag{26}$$

we get

$$\frac{\partial^2}{\partial x^2}\psi_x = -\frac{1}{\hbar^2\ c^2}\ (E_o^2 + E_{p_x}^2)\ \psi_x \tag{27}$$

2.1.3 The space independent differential equation.

We derivate the wave function ψ_x two times versus t

$$\frac{\partial^2}{\partial t^2}\psi_x = -\frac{c^2}{\hbar^2}\ p_x^2\ \psi_x \tag{28}$$

and with

$$E_{p_x} = p_x\ c \qquad and \qquad E_p^2 = E_{p_x}^2 + E_{p_y}^2 + E_{p_z}^2 \tag{29}$$

we get

$$-\ \hbar^2\ \frac{\partial^2}{\partial t^2}\psi_x = E_{p_x}^2\psi_x \tag{30}$$

and for the space

$$-\ \hbar^2\ \Delta_{\mathbf{t}}\psi = E_p^2\psi \tag{31}$$

with the operator $\Delta_{\mathbf{t}}$ defined in sec. 2.4.

2.2 Non relativistic differential equations

For non relativistic speeds we have that $v \ll c$ and that $E_{kin_x} \approx p^2/(2m_o)$.

2.2.1 General non relativistic differential equation.

The general non relativistic differential equation we obtain by deriving ψ_x two times versus t and one time versus x. The results are then connected through $E_{rel_x} - E_o = E_{kin_x} \approx p^2/(2m_o)$. We get

$$-\ i\ \hbar\ c\frac{\partial}{\partial x}\psi_x(x,t)\ -\ E_o\ \psi_x(x,t) \approx\ -\frac{\hbar^2}{2\ m_o\ c^2}\ \frac{\partial^2}{\partial t^2}\psi_x(x,t) \quad with \quad E_o = m_o\ c^2 \tag{32}$$

The differential equation with the constant energy E_o describes the movement of a non-accelerated particle in a cero potential energy field.

With E_{tot} the total energy, E_{kin} the kinetic energy, E_{pot} the potential energy and E_{rel} the relativistic energy, the above equation is equivalent to $E_{rel}\ -\ E_o = E_{kin}$. If we add at to the kinetic energy E_{kin} the potential energy $E_{pot} = U_x(x,t)$ we get the total energy E_{tot} for an accelerated movement. The result is

$$-\ i\,\hbar\,c\frac{\partial}{\partial x}\psi_x(x,t)\ -\ E_o\,\psi_x(x,t)\ +\ U_x(x,t)\psi_x(x,t) = E_{tot}\psi_x(x,t) \tag{33}$$

$$-\ \frac{\hbar^2}{2\,m_o\,c^2}\,\frac{\partial^2}{\partial t^2}\psi_x(x,t)\ +\ U_x(x,t)\psi_x(x,t) = E_{tot}\psi_x(x,t) \tag{34}$$

In a conservative system the total energy is time independent with $E_{tot} = constant$.

Comparing equation (32) with the **General Schrödinger** differential equation, the main difference is that equation (32) derives one time versus space and two times versus time, in other words, time and space are interchanged.

2.2.2 The time independent non relativistic differential equation.

Differential equations are deduced in derivating one time or two times the wave function ψ_x.

a) We derivate the wave function ψ_x one time versus x

$$\frac{\partial}{\partial x}\psi_x = \frac{i}{\hbar\,c}\,E_{rel_x}\,\psi_x = \frac{i}{\hbar\,c}\,(E_o + E_{kin_x})\,\psi_x \tag{35}$$

For a conservative field $U_x = q_e\,V_x$ with a total energy E_{tot_x} we have

$$E_{tot_x} = E_{kin_x} + U_x \qquad and\ \ with \qquad E_{kin_x} \approx \frac{1}{2\,m_o}\,p_x^2 \tag{36}$$

we get

$$\left\{ -\ i\,\hbar\,c\,\frac{\partial}{\partial x} + U(x)\ \right\}\psi(x) \approx E_x\,\psi(x) \tag{37}$$

with

$$E_x = E_{tot_x} + E_o \tag{38}$$

the Eigenvalue.

b) For the time independent differential equation deduced derivating the wave function ψ_x two times versus x see sec. 2.5.

2.2.3 Space independent non relativistic differential equation.

We take two times the derivate of the wave function ψ_x versus t

$$\frac{\partial^2}{\partial t^2}\psi_x = -\ \frac{c^2}{\hbar^2}\,p_x^2\,\psi_x \tag{39}$$

and with eq. (31)

$$- \hbar^2 \Delta_{\mathbf{t}} \psi = E_p^2 \psi \tag{40}$$

and $v \ll c$ and a conservative potential U

$$E_{kin} \approx \frac{1}{2\, m_o}\, p^2 = \frac{E_p^2}{2\, E_o} \qquad and \qquad E_{tot} = E_{kin} + U \tag{41}$$

we obtain the space independent non relativistic differential equation

$$\left\{ - \frac{\hbar^2}{2\, E_o}\, \Delta_{\mathbf{t}} + U \right\} \psi \approx E_{tot}\, \psi \tag{42}$$

which is equivalent to the time inependent equation from Schroedinger.

2.3 Uncertainty principle.

In the proposed model the pairs of canonical conjugated variables lead to the following uncertainty relations

$$(\Delta E) \cdot (\Delta x) \geq \frac{1}{2}\, \hbar\, c \tag{43}$$

and

$$(\Delta p) \cdot (\Delta t) \geq \frac{1}{2}\, \frac{\hbar}{c} \tag{44}$$

Noticeable at this point is the relation

$$E\, r_o = \hbar\, c \tag{45}$$

for a particle, that connects the radius r_o and the relativistic energy E through $\hbar\, c$.

2.4 Operators.

2.4.1 Relativistic operator for the linear momentum.

The relativistic operator for the linear momentum of a particle is

$$\hat{p} = \; i\, \frac{\hbar}{c}\, \frac{\partial}{\partial t} \tag{46}$$

The linear momentum we get with

$$\bar{p}\, \chi = \; i\, \frac{\hbar}{c}\, \nabla_t\, \chi \tag{47}$$

where χ is the total mass-probability function

$$\chi = \psi_x \, \psi_y \, \psi_z \tag{48}$$

and $\nabla_{\mathbf{t}}$

$$\nabla_{\mathbf{t}} = \frac{\partial}{\partial t}|_x \, \mathbf{e}_x \; + \; \frac{\partial}{\partial t}|_y \, \mathbf{e}_y \; + \; \frac{\partial}{\partial t}|_z \, \mathbf{e}_z \tag{49}$$

2.4.2 Relativistic operators for the energy.

For the relativistic energy of a non-accelerated particle we obtain the operator

$$\hat{E}_{rel_x} = - \, i \, \hbar \, c \, \frac{\partial}{\partial x} \tag{50}$$

Application example.
If we apply the relativistic operators to the relativistic energy of a particle

$$E_x^2 = m_o^2 \, c^4 \; + \; p_x^2 \, c^2 \tag{51}$$

we get

$$- \; \hbar^2 \, c^2 \, \frac{\partial^2}{\partial x^2}\psi_x = m_o^2 \, c^4 \, \psi_x \; - \; \hbar^2 \, \frac{\partial^2}{\partial t^2}\psi_x \tag{52}$$

the **Klein-Gordon** equation.
With $m_o = 0$ we have

$$\frac{\partial^2}{\partial x^2}\psi_x \; = \; \frac{1}{c^2} \, \frac{\partial^2}{\partial t^2}\psi_x \tag{53}$$

2.4.3 Non-relativistic operator for the kinetic energy.

The non-relativistic operator for the kinetic energy on the x coordinate is

$$\hat{E}_{kin_x} = - \, \frac{\hbar^2}{2 \, m_o \, c^2} \, \frac{\partial^2}{\partial t^2}|_x \tag{54}$$

and the total kinetic energy E_{kin} in the three dimensional space

$$E_{kin} = E_{kin_x} + E_{kin_y} + E_{kin_z} = - \, \frac{\hbar^2}{2 \, m_o \, c^2} \, \Delta_{\mathbf{t}} \chi \tag{55}$$

with

$$\Delta_{\mathbf{t}} = \frac{\partial^2}{\partial t^2}|_x \; + \; \frac{\partial^2}{\partial t^2}|_y \; + \; \frac{\partial^2}{\partial t^2}|_z \tag{56}$$

2.4.4 Non-relativistic Hamilton operator.

The operator for the non-relativistic total energy on the x coordinate has the form

$$\hat{E}_x = \frac{1}{2\,m_o}\left(\, i\,\frac{\hbar}{c}\,\frac{\partial}{\partial t}|_x\right)^2 \;+\; \hat{U}_x \tag{57}$$

or

$$\hat{E}_x = \frac{\hat{p}_x^{\,2}}{2\,m_o} \;+\; \hat{U}_x \tag{58}$$

which is equal to the Hamilton operator $\hat{H}_x$.
The general non-relativistic differential equation thus takes the form

$$i\;\hbar c\;\frac{\partial}{\partial x}\psi_x(x,t) = \hat{H}_x\;\psi_x(x,t) \tag{59}$$

with

$$\hat{H}_x = \frac{\hat{p}_x^{\,2}}{2\,m_o} \;+\; \hat{U}_x \tag{60}$$

the non-relativistic Hamilton operator.

2.4.5 Non-relativistic operator for the orbital-angular-momentum.

The wave function for the three dimentional space is

$$\psi_x(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\chi(m_{rel})\;exp\left\{i\frac{c}{\hbar}\left[m_{rel}\;\mathbf{r} - \;\mathbf{p}(m_{rel})\;t\right]\right\}dm_{rel} \tag{61}$$

with

$$\mathbf{r} = x\;\mathbf{e}_x + y\;\mathbf{e}_y + z\;\mathbf{e}_z \qquad and \qquad \mathbf{p} = p_x\;\mathbf{e}_x + p_y\;\mathbf{e}_y + p\;\mathbf{e}_z \tag{62}$$

We define the linear momentum operator for the different coordinates as:

$$\hat{p}_k = \;i\;\frac{\hbar}{c}\;\frac{\partial}{\partial t}|_k \tag{63}$$

The orbital-angular-momentum-operator can be expressed as

$$\mathbf{M}\left(\mathbf{r},\; i\;\frac{\hbar}{c}\;\nabla_{\mathbf{t}}\right) = \left(\mathbf{r}\;\times\;i\;\frac{\hbar}{c}\;\nabla_{\mathbf{t}}\right) \tag{64}$$

with

$$\nabla_{\mathbf{t}} = \;\frac{\partial}{\partial t}|_x\;\mathbf{e}_x \;+\; \frac{\partial}{\partial t}|_y\,\mathbf{e}_y \;+\; \frac{\partial}{\partial t}|_z\;\mathbf{e}_z \tag{65}$$

The operators for the vectorcomponents are:

$$\hat{M}_x = \hat{y}\ \hat{p}_z - \hat{z}\ \hat{p}_y \qquad \hat{M}_y = \hat{z}\ \hat{p}_x - \hat{x}\ \hat{p}_z \qquad \hat{M}_z = \hat{x}\ \hat{p}_y - \hat{y}\ \hat{p}_z \tag{66}$$

The conmutations are as known

$$[\hat{M}_k, \hat{M}_{k+1}] \neq 0 \qquad [\hat{M}_k, \hat{Q}] = 0 \qquad with \qquad \hat{Q} = \hat{M}_x^2 + \hat{M}_y^2 + \hat{M}_z^2 \tag{67}$$

2.5 The proposed theory and the Correspondence Principle.

The present theory is based on the radius-energy relation that substitutes the de Broglie wavelength.

The accordance of the proposed theory with the correspondence principle of quantum mechanics is ensured, in that the time independent differential equation from Schroedinger, deduced from the wave package constructed with the de Broglie wavelength, can be derived from the wave package constructed with the radius-energy relation presented in this work.

We start derivating the wave function ψ_x two times versus space, to get the time independent differential equation

$$\frac{\partial^2}{\partial x^2}\psi_x = -\ \frac{c^2}{\hbar^2}\ m_{rel_x}^2\ \psi_x \tag{68}$$

With

$$m_{rel_x} = \frac{1}{c^2}\sqrt{E_o^2 + E_{p_x}^2} \qquad E_o = m_o c^2 \qquad and \qquad E_{p_x} = p_x\ c \tag{69}$$

we get

$$\frac{\partial^2}{\partial x^2}\psi_x = -\ \frac{1}{\hbar^2\ c^2}\ (E_o^2 + E_{p_x}^2)\ \psi_x \tag{70}$$

For non-relativistic velocities $v \ll c$ we have that

$$E_{kin_x} = \frac{p_x^2}{2\ m_o} \qquad and \qquad E_{p_x}^2 = p_x^2\ c^2 = 2\ m_o\ c^2\ E_{kin_x} \tag{71}$$

and we get

$$\frac{\partial^2}{\partial x^2}\psi_x = -\ \frac{2\ m_o}{\hbar^2}\ \left[\frac{1}{2}E_o + E_{kin_x}\right]\ \psi_x \tag{72}$$

With a conservative potential $E_{tot_x} = U_x + E_{kin_x}$ we get finally

$$\left[-\ \frac{\hbar^2}{2\ m_o}\frac{\partial^2}{\partial x^2}\ +\ U_x\right]\psi_x\ =\ E_x\ \psi_x \qquad with \qquad E_x = \frac{1}{2}\left[E_o + 2\ E_{tot_x}\right] \tag{73}$$

For the three dimensional space we have

$$\left[-\frac{\hbar^2}{2\,m_o}\,\Delta_{\mathbf{r}} \;+\; U \right]\,\chi = E\,\chi \tag{74}$$

with $\Delta_{\mathbf{r}}$ the Laplace operator and

$$E = \frac{1}{2}\,[E_o + 2\;E_{tot}] \tag{75}$$

If we make $E_o = 0$ we get

$$\left[-\frac{\hbar^2}{2\,m_o}\,\Delta_{\mathbf{r}} \;+\; U \right]\,\chi = E_{tot}\,\chi \tag{76}$$

Eq. (76) is exactly the time independent differential equation constructed by **Schroedinger** with E_{tot} the Eigenvalue.

2.6 The mass conservation equation.

The mass conservation differential equation we obtain by derivating ψ_x one time versus t and one time versus x. The results are then connected through

$$p_x = \frac{v_x}{c^2} E_{rel_x} \tag{77}$$

We get

$$\frac{\partial}{\partial t}\psi_x(x,t) \;=\; -\,v_x\,\frac{\partial}{\partial x}\psi_x(x,t) \tag{78}$$

We define the mass probability density as

$$\rho_x(x,t) = \psi_x^*(x,t)\;\psi_x(x,t) \qquad or \qquad \rho(\mathbf{r},t) = \psi^*(\mathbf{r},t)\;\psi(\mathbf{r},t) \tag{79}$$

We derive the mass probability density versus time

$$\frac{\partial}{\partial t}\rho_x(x,t) = \frac{\partial}{\partial t}\,[\psi_x^*(x,t)\;\psi_x(x,t)] = \frac{\partial}{\partial t}\psi_x^*(x,t)\;\psi_x(x,t) \;+\; \psi_x^*(x,t)\;\frac{\partial}{\partial t}\psi_x(x,t) \tag{80}$$

With eq. (78) we get

$$\frac{\partial}{\partial t}\rho_x(x,t) = -v_x\left[\frac{\partial}{\partial x}\psi_x^*(x,t)\;\psi_x(x,t) \;+\; \psi_x^*(x,t)\;\frac{\partial}{\partial x}\psi_x(x,t)\right] \tag{81}$$

or

$$\frac{\partial}{\partial t}\rho_x(x,t) = -v_x \frac{\partial}{\partial x}\left[\psi_x^*(x,t)\ \psi_x(x,t)\right] = -\frac{\partial}{\partial x}\left[v_x\ \rho_x(x,t)\right] = -\frac{\partial}{\partial x} j(x,t) \tag{82}$$

or

$$\frac{\partial}{\partial t}\rho(\mathbf{r},t) \ = \ -\nabla_{\mathbf{r}}\,\mathbf{j}(\mathbf{r},t) \qquad with \qquad \mathbf{j}(\mathbf{r},t) = \mathbf{v}\ \psi^*(\mathbf{r},t)\ \psi(\mathbf{r},t) \tag{83}$$

where $\mathbf{j}(\mathbf{r},t)$ is the mass-current probability density.

2.7 The wave equation for relativistic speeds.

We start with the wave eq. (13) from sec. 1.2

$$\psi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \chi_x(m_{rel_x})\ exp\left[i\frac{c}{\hbar}\left(m_{rel_x}\ x - \ p_x(m_{rel_x})\ t\right)\right] dm_{rel_x} \tag{84}$$

and analyze the equation for relativistic speeds where $\Delta v = c - v \ll c$. We get

$$E_{rel} = E_p = p\ c = \frac{m\ v}{\beta}\ c \qquad \beta = \sqrt{1 - \frac{v^2}{c^2}} \qquad \lambda = \frac{h}{p} \tag{85}$$

The resulting wave equation is

$$\psi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \chi_x(m_{rel_x})\ exp\left[\frac{i}{\hbar}\left(p\ x - \ E_{pv}\ t\right)\right] dm_{rel_x} \tag{86}$$

where

$$E_{pv} = p\ v = \frac{m\ v}{\beta}\ v \tag{87}$$

With $E_{rel} = pc^2/v$ and $E_o^2 \ll E_p^2$ we get

$$E_{pv} = p\ v = \frac{p^2\ c^2}{E_{rel}} = \frac{p^2\ c^2}{\sqrt{E_o^2 + E_p^2}} \approx pc = E_p \tag{88}$$

We now derive the wave equation one time versus space and one time versus time and connect the results with $E_{pv} = pc$. We get

$$\frac{\partial}{\partial t}\psi_x = -\ c\ \frac{\partial}{\partial x}\psi_x \tag{89}$$

3 Wave equations for free moving particles.

3.1 The relativistic wave equation for the free moving particle.

Until now we have worked with the wave package defined with eq. (13) where the integration is made versus dm_{rel_x}. In what follows the wave package defined with eq. (10) is used where the integration is made versus dp.

We start with the dispersion equations for the relativistic mass m_{rel_x} of sec. 1.2. In what follows we omit the sub-index x and write m_{rel} instead of m_{rel_x}.

$$m_{rel} = \frac{E_{rel}}{c^2} \qquad m_{rel} = m_{rel}(p) = \frac{1}{c^2}\sqrt{E_o^2 + p^2c^2} \qquad and \qquad E_o = m_o c^2 \tag{90}$$

which can be transformed to

$$m_{rel} = \frac{1}{c}\left[p^2 + \frac{E_o^2}{c^2}\right]^{1/2} = \frac{1}{c}\ \left[p + p^{'}\right] \tag{91}$$

with

$$p^{'}_{1,2} = -p \pm \sqrt{p^2 + \frac{E_o^2}{c^2}} \tag{92}$$

We also transform

$$p(m_{rel}) = c\ \sqrt{m_{rel}^2\ -\ m_o^2} \qquad and \qquad m_o = \frac{E_o}{c^2} \tag{93}$$

to

$$p = \frac{1}{c}\left[E_{rel}^2 - m_o^2\ c^4\right]^{1/2} \qquad with \qquad E_{rel} = E_o + E_{kin} \tag{94}$$

and

$$p = \frac{1}{c}\left[E_{kin}^2 + 2\ E_o\ E_{kin}\right]^{1/2} = \frac{1}{c}\left[E_{kin} + E^{'}\right] \tag{95}$$

with

$$E^{'}_{1,2} = -E_{kin} \pm \sqrt{E_{kin}^2 + 2\ E_o\ E_{kin}} \tag{96}$$

Note: In what follows we changed the symbol for the wave function from ϕ to Ψ to follow the convention.

If we now introduce (91) and (95) in eq. (10)

$$\Psi(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \kappa_x(p_x)\ exp\left\{i\frac{c}{\hbar}\left[m_{rel_x}(p_x)\ x -\ p_x\ t\right]\right\} dp_x \tag{97}$$

we get

$$\Psi(x,t) \propto \ exp\left\{\frac{i}{\hbar}\left[\ [p+p^{'}]x-[E_{kin}+E^{'}]t\ \right]\right\} \tag{98}$$

what we can write in the form

$$\Psi(x,t) \propto \ exp\left\{\frac{i}{\hbar}\left[p^{'}x-E^{'}t\ \right]\right\}\cdot exp\left\{\frac{i}{\hbar}\left[p\ x-E_{kin}\ t\ \right]\right\} \tag{99}$$

We know that

$$E_{rel}=E_o\ +\ E_{kin}=E_s\ +\ E_n \tag{100}$$

with

$$E_s=\frac{E_o^2}{\sqrt{E_o^2+E_p^2}} \qquad E_n=\frac{E_p^2}{\sqrt{E_o^2+E_p^2}} \qquad E_p=p\ c \tag{101}$$

For *relativistic speeds* $v>0.95c$ we have that

$$E_s << E_n \qquad E_{rel}\approx E_n\approx E_p \qquad E_{kin}\approx E_n\ -\ E_o \tag{102}$$

and

$$p_1^{'}=0 \qquad p_2^{'}=-2p \qquad E_1^{'}=0 \qquad E_2^{'}=-2E_{kin} \tag{103}$$

and get

$$\Psi(x,t) \propto \ exp\left\{\pm\frac{i}{\hbar}\left[p\ x-E_{kin}\ t\ \right]\right\}=\ exp\left\{\pm\frac{i}{\hbar}\left[p\ x-(E_n-E_o)\ t\ \right]\right\} \tag{104}$$

where E_{kin} is the relativistic kinetic energy.

3.1.1 The wave package for the relativistic wave equation.

To get the wave package we derive (104) one time versus space and one time versus time.

$$c\ \frac{\partial}{\partial x}\psi_x\ \propto\ \pm\ \frac{i}{\hbar}\ p\ c\ \psi_x \tag{105}$$

$$\frac{\partial}{\partial t}\psi_x\ \propto\ \pm\ \frac{i}{\hbar}\left[p\ c\ -\ E_o\right]\psi_x \tag{106}$$

We now eliminate from the two equations $p\,c\,\psi_x$ and get

$$\frac{\partial}{\partial t}\psi_x \;\propto\; -\; c\,\frac{\partial}{\partial x}\psi_x \;\pm\; \frac{i}{\hbar}\,E_o\,\psi_x \tag{107}$$

The time independent equation is

$$-\,i\,\hbar\,c\,\frac{\partial}{\partial x}\psi_x \;=\; \pm\,E_o\,\psi_x \tag{108}$$

which with an potential $U(x)$ gives

$$-\,i\,\hbar\,c\,\frac{\partial}{\partial x}\psi_x \;+\; U(x)\psi_x = \;[\pm\,E_o \;+\; E_{tot}]\;\psi_x = E\psi_x \tag{109}$$

If we compare it with (33) which was derived with the wave package defined with eq. (13) where the integration is made versus dm_{rel_x}, and which was derived as non relativistic

$$-\;\;i\,\hbar\,c\frac{\partial}{\partial x}\psi_x(x,t)\;-\;E_o\,\psi_x(x,t)\;+\;U_x(x,t)\psi_x(x,t) = E_{tot}\psi_x(x,t) \tag{110}$$

we see that they are equal. This means that we have the same equation for non relativistic and relativistic problems.

3.2 The slightly relativistic wave equation for the free moving particle.

For $v << c$ we have that $p \approx mv$

$$E_s \approx E_o \qquad and \qquad E_n \approx E_{rel} - E_o = E_{kin} \tag{111}$$

Also for $v \to 0$ we get that

$$E_{kin} \to 0 \qquad and \qquad E^{'} \to 0 \qquad for \qquad v \to 0 \tag{112}$$

and

$$p \to 0 \qquad and \qquad p^{'} \to\; mc \qquad for \qquad v \to 0 \tag{113}$$

From (99) we get

$$\Psi(x,t) \propto\; exp\left\{\frac{i}{\hbar}\,[mc\,x\,]\right\}\cdot exp\left\{\frac{i}{\hbar}\,[p\,x - E_{kin}\,t\,]\right\} \tag{114}$$

where we have that the first exponent is not a function of p and E_{kin}. As $p = mv$ from the second exponent is much smaller than mc from the first exponent, the first

exponent oscillates along the $x - axis$ between plus and minus of its absolute value which is one. The frequency of the oscillation of the first factor is very high compared with the second, and the first factor can be made equal to one for all x.

$$\Psi(x,t) \propto \; exp\left\{ \frac{i}{\hbar} \left[p\; x - E_{kin}\; t \; \right] \right\} \tag{115}$$

With $p \approx mv$ we also can write

$$E_{kin} \approx -\frac{c^2}{2E_o}\; p^2 \; + \; \frac{1 \cdot 3}{2 \cdot 4}\; \frac{c^4}{E_o^3}\; p^4 \; - \; \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\; \frac{c^6}{E_o^5}\; p^6 + \cdots \tag{116}$$

and arrive to the relativistic wave equation for a free moving particle

$$i\; \hbar\; \frac{\partial}{\partial t} \Psi = \; \left[\; \frac{\hbar^2}{2m}\; \frac{\partial^2}{\partial x^2} \; + \; \frac{1 \cdot 3}{2 \cdot 4}\; \frac{\hbar^4}{m^3 c^2}\; \frac{\partial^4}{\partial x^4} \cdots \right] \; \Psi \tag{117}$$

If we take into consideration only the first two terms of E_{kin} and introduce an external potential $U(x)$, we get the following time independent wave equation for a slightly relativistic moving charged particle in an external potential.

$$\left[\; \frac{\hbar^2}{2m}\; \frac{\partial^2}{\partial x^2} \; + \; \frac{1 \cdot 3}{2 \cdot 4}\; \frac{\hbar^4}{m^3 c^2}\; \frac{\partial^4}{\partial x^4} \; + \; U(x) \right] \; \Psi = E\; \Psi \tag{118}$$

To calculate the maximum velocity v_{max} for this case we make the third term of eq. (116) ten times smaller than the second term and get $v_{max} = 0.346\; c$. It is not recommended to use more than two terms of eq. 116 because of the approximations made for the deduction.

Note: Eq. 118 allows to calculate the solutions for QM systems which are slightly relativistic instead of using the strong relativistic Dirac formulation.

3.3 The non-relativistic wave equation for the free moving particle

If we make $E_o = 0$ because we want an equation that describes only the kinetic energy we get $p^{'} = 0$ and $E^{'} = 0$, and if we reduce our observation to non-relativistic speeds with $v << c$ we have from eq. (99)

$$\Psi(x,t) \propto exp\left\{ \frac{i}{\hbar} \left[p\; x - E_{kin}\; t \; \right] \right\} \qquad with \qquad E_{kin} = \frac{1}{2}\; \frac{p^2}{m} = E_{kin}(p) \tag{119}$$

$$\Psi(x,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \kappa_x(p_x)\; exp\left\{ \frac{i}{\hbar} \left[p\; x - E_{kin}(p)\; t \right] \right\} dp_x \tag{120}$$

The wave function derived two times versus x and one time versus t gives the

differential equation of the free moving particle of mass m. If we introduce an external potencial U we have the Schrödinger equation for an accelerated particle.

$$i\,\hbar\,\frac{\partial}{\partial t}\Psi(x,t)\;\approx\left[-\frac{\hbar^2}{2\,m_o}\,\frac{\partial^2}{\partial x^2}+U\right]\Psi(x,t) \tag{121}$$

4 Applications of the non-relativistic differential equation

The solutions of the time independent non-relativistic differential equation (33) for a potential pot, the harmonic oscillator and the hydrogen atom are derived.

4.1 Potential pot

The non-relativistic time independent differential equation is

$$-\;i\,\hbar\,c\,\frac{\partial}{\partial x}\psi_x(x)\;+\;U_x(x)\,\psi_x(x)=[E_{tot}\;+\;E_o]\,\psi_x(x)=E\,\psi_x(x) \tag{122}$$

With $y=\psi_x(x)$ we can write

$$-\;i\,\hbar\,c\,\frac{dy}{y}=[E-U]\,dx \tag{123}$$

After integration we get

$$-\;i\,\hbar\,c\;[\ln|y|+\ln C_y]=\int[E-U]\,dx \tag{124}$$

resulting

$$|y|=\frac{1}{C_y}\,exp\left\{\frac{i}{\hbar\,c}\int[E-U]\,dx\right\} \tag{125}$$

Equation (125) is valid for all potential energies U and gives real values for y if

$$\left\{\frac{i}{\hbar\,c}\int[E-U]\,dx\right\}=k\,\pi\qquad and\qquad k=0,\;\pm1,\;\pm2,\;\pm3,\cdots \tag{126}$$

defining the quantization condition, which together with the normalization condition allows the calculation of the eigenfunctions.

The potential pot is defined as

$$U = \begin{cases} \infty & \text{for } x \leq 0 \\ 0 & \text{for } 0 < x < a \\ \infty & \text{for } x \geq a \end{cases}$$

and we have for $U = 0$ and a constant E because of the assumption of energy conservation

$$\frac{1}{\hbar\, c}\, E\, x = k\, \pi \qquad \textit{resulting with } x = a \qquad E_k = \pi\, \frac{\hbar\, c}{a}\, k \tag{127}$$

with $k = 0,\ \pm 1,\ \pm 2,\ \pm 3, \cdots$ the eigenvalues E_k.

The total energy is with $E_k = E_{tot} + E_o$

$$E_{tot} = E_k - E_o = \pi\, \frac{\hbar\, c}{a}\, k\ -\ E_o \tag{128}$$

and for $E_{tot} = 0$ we get

$$a_o = k\, \frac{\pi \hbar\, c}{E_o} = k\, \pi\, r_o \qquad \textit{with} \qquad \frac{\hbar\, c}{E_o} = r_o \tag{129}$$

the radius of of a rest electron or positron.

The eigenfunction is

$$y_k = \frac{1}{C_y}\, exp\left\{ \frac{i}{\hbar\, c}\, E_k\, x \right\} \tag{130}$$

The integration constant C_y we get with the normalization condition

$$\int_{-\infty}^{\infty} y_{k'}^{*}\ y_k\, dx = \delta_{(k',k)} \tag{131}$$

For $k^{'} = k$ we get

$$\frac{1}{C_y^2}\ \int_0^a exp\left\{ \frac{i}{\hbar\, c}\, [E_{k'} - E_k]\, x \right\}\, dx = 1 \tag{132}$$

resulting

$$\frac{1}{C_y^2}\ = a \qquad or \qquad C_y = \sqrt{a} \tag{133}$$

The normalized eigenfunction is

$$y_k = \frac{1}{\sqrt{a}}\, exp\left\{ \frac{i}{\hbar\, c}\, E_k\, x \right\} \tag{134}$$

Conclusion: The main differences compared with the solution obtained with the Schroedinger equation is that the quantization of the energy E_k is proportional to k instead of k^2 and for defined values of a the total energy E_{tot} becomes zero.

4.2 Harmonic oscillator

The potential energy for the harmonic oscillator is

$$U(x) = \frac{K}{2}\, x^2 = \frac{m\,\omega^2}{2}\, x^2 \qquad with \qquad \omega^2 = K/m \tag{135}$$

With eq. (125) we get

$$|y| = \frac{1}{C_y}\, exp\left\{ \frac{i}{\hbar\, c} \int \left[E - \frac{K}{2}\, x^2 \right] dx \right\} \tag{136}$$

With the quantization condition we get

$$\frac{1}{\hbar\, c} \int_0^a \left[E - \frac{K}{2}\, x^2 \right] dx = \frac{1}{\hbar\, c} \left[E\, a - \frac{K}{6}\, a^3 \right] = k\, \pi \tag{137}$$

resulting for the quantized energy with $E_{tot} = E_k - E_o$

$$E_{tot} = \pi\, \frac{\hbar\, c}{a} \left[k \,+\, \frac{1}{6}\, \frac{m\,\omega^2}{\pi\, \hbar\, c}\, a^3 \right] - E_o = E_k - E_o \tag{138}$$

The minimum quantum change between two adjacent energy levels is

$$\Delta E_{tot} = \Delta E_k = \pi\, \frac{\hbar\, c}{a} \tag{139}$$

For $E_{tot} = 0$ we get

$$a \left[E_o - \frac{1}{6}\, m\, \omega^2\, a^2 \right] = k\, \pi\, \hbar\, c \tag{140}$$

which for $k = 0$ gives

$$a_1 = 0 \qquad or \qquad a_{2,3} = \pm\sqrt{\frac{6\, E_o}{m\, \omega^2}} \qquad for \qquad k = 0 \tag{141}$$

We get for the minimum quantum change between two adjacent energy levels

$$\Delta E_{tot} = \pm\, \frac{\pi}{\sqrt{6}}\, \hbar\omega \tag{142}$$

The minimum quantum energy difference ΔE_{tot} between two adjacent energy levels is proportional to $\hbar\omega$.

With the normalization condition given by equation (131) we have that

$$\int_{-\infty}^{\infty} y_{k'}^*\, y_k\, dx = \frac{1}{C_y^2} \int_{-\infty}^{\infty} exp\left\{ \frac{i}{\hbar\, c}\, [E_{k'} - E_k]\, x \right\} dx \tag{143}$$

or

$$\frac{\hbar\, c}{C_y^2}\ \int_{-\infty}^{\infty} exp\,\{\ i\ [E_{k'} - E_k]\ \eta\}\ d\eta = \frac{\hbar\, c}{C_y^2}\ \delta_{(k',k)} \quad with \quad \eta = \frac{x}{\hbar\, c} \tag{144}$$

With $k^{'} = k$ we get the integration constant $C_y = \sqrt{\hbar\, c}$ resulting the normalized eigenfunctions

$$y_k = \frac{1}{\sqrt{\hbar\, c}}\ exp\left\{\ \frac{i}{\hbar\, c}\ \left[E_k\ x - \frac{K}{6}\ x^3\right]\right\} \tag{145}$$

4.3 Hydrogen atom

We start with the deduction of the quantization conditions from eq. (33) which was deduced for non relativistic speeds but is also valid for relativistic speeds as shown in sec. 3.1.1.

$$-\ i\ \hbar\, c\ \frac{\partial}{\partial x}\psi_x(x)\ +\ U_x(x)\ \psi_x(x) = [E_o\ +\ E_{tot}]\ \psi_x(x) = E\ \psi_x(x) \tag{146}$$

which is equivalent to

$$E_{rel} + U = E_o + E_{kin} + U = E \qquad E_{tot} = E_{kin} + U \qquad E_{rel} = E_o + E_{kin} \tag{147}$$

We define the operator

$$\vec{\nabla}\ \cdot\ \vec{E} = \nabla E = \frac{\partial}{\partial x}\ +\ \frac{\partial}{\partial y}\ +\ \frac{\partial}{\partial z} \quad with \quad \vec{E} = \vec{e}_x\ +\ \vec{e}_y\ +\ \vec{e}_z \tag{148}$$

$$\nabla E\ \psi(x,y,z) = \frac{\partial}{\partial x}\psi(x,y,z)\ +\ \frac{\partial}{\partial y}\psi(x,y,z)\ +\ \frac{\partial}{\partial z}\psi(x,y,z) \tag{149}$$

For polar coordinates we write

$$-\ i\ \hbar\, c\ \nabla\ \chi(r,\theta,\varphi)\ +\ U\ \chi(r,\theta,\varphi) = E\ \chi(r,\theta,\varphi) \tag{150}$$

with the ∇ operator expressed in polar coordinates

$$\nabla = \frac{\partial}{\partial r}\ +\ \frac{2}{r}\ +\ \frac{1}{r\ \sin\theta}\ \frac{\partial}{\partial\varphi}\ +\ \frac{1}{r}\ \frac{\partial}{\partial\theta}\ +\ \frac{1}{r}\ \cot\theta \tag{151}$$

The differential equation has now the form

$$\left[\ \nabla\ +\ \frac{i}{\hbar\, c}\ U\ \right]\ \chi = \frac{i}{\hbar\, c}\ E\ \chi \tag{152}$$

We now assume that the wave function χ can be expressed as a product of a function exclusively of the distance r and a function of the angular variables θ and φ.

$$\chi(r,\theta,\varphi) = R(r)\; Y(\theta,\varphi) \tag{153}$$

We get

$$\left[\frac{d}{dr} + \frac{4}{r}\right] R\cdot Y + \frac{1}{r}\;\Lambda Y\cdot R + \frac{i}{\hbar\, c} U\cdot R\cdot Y = \frac{i}{\hbar\, c} E\cdot R\cdot Y \tag{154}$$

with the operator Λ

$$\Lambda = \frac{1}{\sin\theta}\;\frac{\partial}{\partial\varphi}\;+\;\frac{\partial}{\partial\theta}\;+\;2\;\cot\theta \tag{155}$$

We now assume that

$$\Lambda Y = -\lambda Y \tag{156}$$

and get two separate differential equations for $R(r)$ and $Y(\theta,\varphi)$.

$$\frac{d}{dr}R - \frac{i}{\hbar\, c}\left[E-U\right]R + \frac{1}{r}\left[4-\lambda\right]R = 0 \tag{157}$$

and

$$\left[\frac{1}{\sin\theta}\;\frac{\partial}{\partial\varphi}\;+\;\frac{\partial}{\partial\theta}\;+\;2\;\cot\theta\right] Y = -\lambda Y \tag{158}$$

After multiplying Eq. (157) with dr/R and integrating we get

$$\ln R = \frac{i}{\hbar\, c}\int_{r_u}^{r}\left[E-U\right]dr - \left[4-\lambda\right]\ln\frac{r}{r_u} \tag{159}$$

where r_u and r are arbitrary integrating limits that will be defined later on.

From the solution of eq. (158) results that $\lambda = i\;l$ with $l = \;0,\;\pm1,\;\pm2;\cdots$ as will be shown later at sec. 4.3.2. We get

$$R = \;exp\left\{-4\;\ln\frac{r}{r_u}\right\}\;exp\left\{\frac{i}{\hbar\, c}\left[\int_{r_u}^{r}(E-U)dr + l\;\hbar\, c\;\ln\frac{r}{r_u}\right]\right\} \tag{160}$$

The quantization condition requires that

$$\frac{1}{\hbar\, c}\left[\int_{r_u}^{r}(E-U)dr + l\;\hbar\, c\;\ln\frac{r}{r_u}\right] = k\;\pi \quad with \quad k = \;0,\;\pm1,\;\pm2;\cdots \tag{161}$$

Equation (161) is valid for all point symmetrical potentials U. We now introduce the potential of an atomic nucleus

$$U = -\,Z\,\frac{K_u}{r} \qquad with \qquad K_u = \frac{e^2}{4\pi\,\epsilon_o} \tag{162}$$

Note: According to the focal-point approach, nuclei are composed of electrons and positrons that neither attract nor repel each other for the distance between them tending to zero.

If N_p are the number of positrons and N_e the number of electrons which constitute the nucleus we have that

$$Z = N_p\ -\ N_e \tag{163}$$

For the hydrogen it is $N_p = 919$ and $N_e = 918$.

For energy conservation conditions we have that

$$\int_{r_u}^{r} E dr = E\,(r - r_u) \tag{164}$$

with the value E a constant. We get

$$E = \left[\,k\,\pi\,\hbar\,c - (Z\,K_u + l\,\hbar\,c)\,\ln\frac{r}{r_u}\right]\,\frac{1}{r - r_u} \tag{165}$$

In eq. (165) the terms represent $E = \bar{E}_k + \bar{U} + \bar{E}_l$ where

$$\bar{E}_k = \frac{k\,\pi\,\hbar\,c}{r - r_u} \qquad \bar{U} = -\,\frac{Z\,K_u}{r - r_u}\ln\frac{r}{r_u} \qquad \bar{E}_l = -\,\frac{l\,\hbar\,c}{r - r_u}\ln\frac{r}{r_u} \tag{166}$$

To arrive to the Balmer equation for the hydrogen atom the following steps are necessary.

Step one:

The term that describes the potential energy

$$\bar{U} = -\,\frac{Z\,K_u}{r - r_u}\ln\frac{r}{r_u} = -\,\frac{Z\,e^2}{4\pi\,\epsilon_o}\,\frac{1}{r - r_u}\ln\frac{r}{r_u} \tag{167}$$

gives the potential energy $\bar{U}$ for an orbital electron and Z charges e^+ at the atomic nucleus.

We now assume, that the orbital electron can interact with n_p positrons of the N_p positrons of the nucleus, where $n_p >= Z$.

$$\bar{U}_n = -\,\frac{n_p\,e^2}{4\pi\,\epsilon_o}\,\frac{1}{r - r_u}\ln\frac{r}{r_u} \tag{168}$$

The concept is shown in Fig. 1

Step two:

As the radius $r^{'}$ of an atom is constant, the potential energy is constant for all

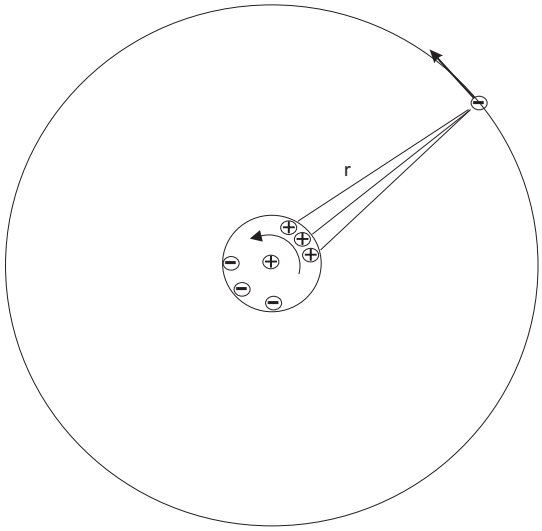

Figure 1: Orbital electron with $n_p = 3$.

number n_p of positrons the orbital electron can interact. We can write

$$\bar{U}_n = -\, K_u\, \frac{n_p}{r - r_u} \ln \frac{r}{r_u} = -\, \frac{e^2}{4\pi\, \epsilon_o}\, \frac{1}{r'} = -\, \frac{K_u}{r'} \qquad\qquad n_p >= Z \tag{169}$$

We get that

$$\frac{1}{r - r_u}\ \ln \frac{r}{r_u} = \frac{1}{n_p\, r'} \qquad\qquad r' = constant \tag{170}$$

Step three:

From eq. (167) we get

$$\bar{U} = -\, Z\, \frac{K_u}{r - r_u} \ln \frac{r}{r_u} = -\, Z\, \frac{K_u}{n_p\, r'} \tag{171}$$

If we now assume that the quantization of the charges of the nucleus which interact with the orbital electron follows the rule $n_p = n^2$, we get for the energy levels

$$\bar{U} = -\, Z\, \frac{K_u}{n^2\ r'} \qquad n = 1,\ 2,\ 3, \cdots \qquad\qquad n_p = n^2 \tag{172}$$

The energy levels of the orbital electron have their origin in the number of positrons n_p of the nucleus with which they interact. The number is given by the quantum number n. We have for

$$n = 1,\ 2,\ 3,\ 4 \qquad respectively \qquad n_p = 1,\ 4,\ 9,\ 16 \tag{173}$$

The difference between energy levels is

$$\Delta\bar{U} = Z\ \frac{K_u}{r^{'}}\ \left[\frac{1}{n^2} - \frac{1}{(n+\Delta n)^2}\right] \qquad \Delta n = 0,\ 1,\ 2, \cdots \tag{174}$$

For $\Delta n = 1$ we get

$$\Delta\bar{U} = Z\ \frac{K_u}{r^{'}}\ \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \tag{175}$$

which for $Z = 1$ is equal to Balmers spectroscopic equation for the hydrogen, namely

$$E = h\ c\ R_H\ \frac{1}{n^2} \qquad and \qquad \Delta E = h\ c\ R_H\ \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \tag{176}$$

with R_H the Rydberg constant and $n = 1,\ 2, ...$

From the two equations (175) and (176) for the potential energy we get

$$\frac{K_u}{r^{'}_H} = h\,c\,R_H \qquad r^{'}_H = \frac{K_u}{h\,c\,R_H} = 1.05811 \cdot 10^{-10}\ \ m \tag{177}$$

The relation between the **mean** distance $r^{'}_H$ and the Bohr radius a_o is

$$r^{'}_H = 2\ a_o = 1.05811 \cdot 10^{-10}\ \ m \tag{178}$$

We conclude, that the potential levels of the orbital electron at the hydrogen atom have their origin in the number of positrons of the nucleus that interact with the orbital electron. From the 919 positrons of the hydrogen nucleus, at each potential level $n_p = n^2$ interact with the orbital electron.

The proposed approach " Emission & Regeneration" UFT is based on focal-point representation of subatomic particles. Electrons and positrons are represented as focal-points of rays of Fundamental Particles (FPs) that move from infinite to infinite with light speed or infinite speed. A focal-point emits FPs with light speed and is regenerated by FPs with infinite speed and vice-versa. There are two types of electrons and positrons according they emit FPs with light (deccelerating=dec) or with infinite (accelerating=acc) speed. Acceleration or deceleration refers to the speed of the outgoing FPs relative to the incoming FPs at the focal-point. Lets call them

- acc^+ positron that emits FPs with infinite speed
- dec^+ positron that emits FPs with light speed
- acc^- electron that emits FPs with infinite speed
- dec^- electron that emits FPs with light speed

In the proposed approach electrons and positrons don't have an **intrinsic** spin. The spin has its origin in a circular movement of the focal point on the orbit of the electron

similar to the movement of an epysicle. See sec. 5.2.

The infinite speed for FPS is a requirement that comes from the need that subatomic particles must be regenerated immediately after having emitted FPs. The infinite speed also explains entanglement.

Regenerating FPs of subatomic particles are those FPs that have been emitted previously by other subatomic particles. All existing electrons and positrons are connected through their rays of emitted and regenerating FPs.

4.3.1 Generalization of the procedure to derive the splitting of the energy levels

From the previous steps required to derive the splitting of the potential energy, we now establish the general rule to derive the splitting of the energies of the orbital electrons.

The rule is as follows:

With a term of the type

$$B = A\,\frac{\ln\frac{r}{r_u}}{r-r_u} \tag{179}$$

where r and r_u are arbitrary integration limits, we can build an equation with a constant radius $r^{'}$ of the type

$$B^{'}_{\gamma} = A\,\gamma\,\frac{\ln\frac{r}{r_u}}{r-r_u} = \frac{A}{r^{'}} \qquad what\ \ gives \qquad \frac{\ln\frac{r}{r_u}}{r-r_u} = \frac{1}{\gamma\,r^{'}} \tag{180}$$

If we introduced the result in eq. 179 we get

$$B = A\,\frac{\ln\frac{r}{r_u}}{r-r_u} = \frac{A}{\gamma\,r^{'}} \tag{181}$$

We start applying the rule to the term of the **potential energy** to show that we arrive to the same eq. (172) which led to the Balmer equation. We start with

$$\bar{U} = -K_u\ \frac{\ln\frac{r}{r_u}}{r-r_u} \tag{182}$$

We introduce to the equation the factor $\gamma = n^2$ and impose that it must be equal to $K_u/r^{'}$.

$$\bar{U}^{'} = -K_u\,n^2\,\frac{\ln\frac{r}{r_u}}{r-r_u} = -\frac{K_u}{r^{'}} \quad what\ \ gives \quad \frac{\ln\frac{r}{r_u}}{r-r_u} = \frac{1}{n^2\,r^{'}} \tag{183}$$

and with eq. (182)

$$\bar{U} = -\,\frac{K_u}{n^2\,r^{'}} \tag{184}$$

which is equal to eq. (172) which led us to the Balmer equation except for the factor Z.

Now we calculate the splitting of the energy also for the orbital angular momentum quantum number l.

We start with

$$E = K_u \ \frac{\ln \frac{r}{r_u}}{r - r_u} \tag{185}$$

and with eq. (165) with a potential $n_p >= Z$

$$E^{'} = \left[-n_p \ K_u - l \ \hbar \ c \ \ + \ \ \frac{k \ \pi}{\ln \frac{r}{r_u}} \ \hbar \ c \right] \ \frac{\ln \frac{r}{r_u}}{r - r_u} \tag{186}$$

and apply the rule to eq. (186) that we can write with $K_u = \alpha \ \hbar \ c$

$$E^{'} = K_u \ \left[-n_p - \frac{l}{\alpha} \ + \ \ \frac{k \ \pi}{\alpha \ \ln \frac{r}{r_u}} \right] \ \frac{\ln \frac{r}{r_u}}{r - r_u} \ = \ \frac{K_u}{r^{'}} \qquad K_u = \alpha \ \hbar \ c \tag{187}$$

with $\alpha = \frac{1}{137}$ the fine-structure constant.

We get

$$\frac{\ln \frac{r}{r_u}}{r - r_u} = \ \frac{1}{r^{'} \ \left[-n_p - \frac{l}{\alpha} \ + \ \ \frac{k \ \pi}{\alpha \ \ln \frac{r}{r_u}} \right]} \tag{188}$$

and with eq. 185 we get that

$$E = \ \frac{K_u}{r^{'} \ \left[-n_p - \frac{l}{\alpha} \ + \ \ \frac{k \ \pi}{\alpha \ \ln \frac{r}{r_u}} \right]} \tag{189}$$

and with $n_p = n^2$ we get

$$E = \ - \frac{K_u}{r^{'} \ \left[n^2 + \frac{1}{\alpha} \ \left(l \ + \ \ \frac{k \ \pi}{\ln \frac{r}{r_u}} \right) \right]} \tag{190}$$

If we make $k = 0$ we get

$$E^{'} = \ - \ \frac{K_u}{r^{'} \ [n^2 + l \ \alpha^{-1} \ ]} \ = \ - \ \frac{K_u}{r^{'} \ [n^2 + 137 \ l \ ]} \tag{191}$$

With $l = 0$ we get again Balmers equation

Now we calculate $\ln \frac{r}{r_u}$ from eq. 160

$$R(r) = \exp \left(-4 \ \ln \frac{r}{r_u} \right) \qquad k = 0, \pm 1, \pm 2, \cdots \tag{192}$$

For the hydrogen atom it is $R = r_H^{'} = 2\ a_o = 1.06 \cdot 10^{-10}\ m$ we get $\ln \frac{r}{r_u} = 5.74$ what gives

$$\frac{k\ \pi}{\ln \frac{r}{r_u}} = 0.547\ k \approx \frac{1}{2}\ k \qquad k = 0, \pm 1, \pm 2, \cdots \tag{193}$$

We see that the total orbital angular momentum quantum number is

$$j = l\ +\ 0.547\ k\ \approx\ l\ +\ \frac{1}{2}\ k \qquad with \qquad k = 0, \pm 1, \pm 2, \pm 3, \cdots \tag{194}$$

The spectroscopic energy is given by

$$\Delta E = \frac{K_u}{r^{'}} \left\{ \frac{1}{\left[n^2 + \frac{1}{\alpha}\ \left(l\ +\ \frac{k\ \pi}{\ln(r/r_u)} \right) \right]}\ -\ \frac{1}{\left[n^{'\,2} + \frac{1}{\alpha}\ \left(l^{'}\ +\ \frac{k^{'}\ \pi}{\ln(r/r_u)} \right) \right]} \right\} \tag{195}$$

where

$$\ln(r/r_u) = \ -\ \frac{1}{4} \ln R(r) \qquad with \qquad R(r) = r^{'}\ \ the\ atomic\ radius \tag{196}$$

As electrons repel each other they place themselves as far as possible on the orbit. The orbit can be occupied only by two electrons which are placed at the opposite sides of the diameter of the orbit, which is now characterized by the quantum number $k = \pm 1$. This quantum number replaces the fictitious spin $s = \pm 1/2$. The Pauli principle refers now to the following quantum numbers $n,\ l,\ m_l,\ k$ which cannot be all equal for two orbital electrons.

Configuration of electrons

n	**l**	**ml**	**k**	**Electr. per shell**
1	0	0	± 1	2
2	0	0	± 1	2
	1	1, 0, −1	± 1	6
3	0	0	± 1	2
	1	1, 0, −1	± 1	6
	2	2, 1, 0, −1, −2	± 1	10

Note: The present approach gives the principal quantum number a physical interpretation, namely, the number of positrons n_p that interact with the orbital electron.

4.3.2 Deduction of the condition $\lambda = i\ l$.

Now we deduce the condition $\lambda = i\ l$ introduced previously in eq. (158).

$$\left[\frac{1}{\sin\theta}\ \frac{\partial}{\partial\varphi}\ +\ \frac{\partial}{\partial\theta}\ +\ 2\ \cot\theta\right] Y = -\lambda Y \tag{197}$$

We assume that

$$Y(\theta,\phi) = \Theta(\theta)\ \Phi(\varphi) \qquad and \qquad \frac{d}{d\varphi}\Phi = m\ \Phi \tag{198}$$

and with $\Phi(\varphi) = \Phi(\varphi + 2\pi)$ we get

$$\Phi = exp\{m\ \varphi\} \qquad with \qquad m = i\ m_l \qquad and \qquad m_l = \pm 0,\ \pm 1,\ \pm 2; \cdots \tag{199}$$

With eq. (198) we have that eq. (158) transforms to

$$\frac{m}{\sin\theta}\ \Theta + \frac{d}{d\theta}\Theta\ +\ 2\ \cot\theta\ \Theta = -\ \lambda\ \Theta \tag{200}$$

and

$$\frac{d\Theta}{\Theta} = -\ \left[\ \frac{m}{\sin\theta}\ +\ 2\ \cot\theta\ +\ \lambda\ \right]\ d\theta \tag{201}$$

which gives the solution

$$\Theta = \frac{1}{C_\Theta}\ exp\left\{-\int\left[\ \frac{i\ m_l}{\sin\theta}\ +\ 2\ \cot\theta\ +\ \lambda\ \right] d\theta\ \right\} \tag{202}$$

With $\Theta(\theta) = \Theta(\theta + 2\pi)$ we conclude that

$$\Theta = \frac{1}{C_\Theta}\ exp\left\{-2\ln\sin\theta\right\}\ exp\left\{-i\ \left[\ m_l \ln(\csc\theta - \cot\theta)\ +\ l\ \theta\ \right]\right\} \tag{203}$$

with $\lambda = i\ l$ and $l = \pm 0,\ \pm 1,\ \pm 2; \cdots$ what we have anticipated for eq. (160).
Eq.(200) we can now write as

$$\frac{d}{d\theta}\Theta\ +\ i\ \frac{m_l}{\sin\theta}\ \Theta\ =\ -\ 2\ \cot\theta\ \Theta\ -\ i\ l\ \Theta \tag{204}$$

In this equation the real and the imaginary terms must be equal, and we get from the imaginary terms that

$$\frac{m_l}{l} = -\sin\theta \qquad with \qquad m_l = \pm 0,\ \pm 1,\ \pm 2; \cdots \quad and \quad l = \pm 0,\ \pm 1,\ \pm 2; \cdots \tag{205}$$

We conclude, that the relation between the orbital quantum number l and the magnetic quantum number m_l is

$$\left|\frac{m_l}{l}\right| = |-\sin\theta| \leq 1 \qquad or \qquad |m_l| \ \leq \ |-l\sin\theta| \tag{206}$$

m_l is the projection of l on the $x-y$ plane and gives the projection of the orbital area $A = \pi\ l^2$ on the $x-y$ axis.

$$A_{x,y} = \pi\ m_l^2 = \pi\ (l\ \sin\theta)^2 \qquad m_l \leq l \tag{207}$$

$A_{x,y}$ is the part of the orbital area perpendicular to the $z - axis$. The $z - axis$ defines the magnetic flux Φ for an external magnetic field in z direction.

$$\Phi = \vec{B}_z \cdot \vec{A} \qquad \Phi = B_z\ A_z \tag{208}$$

An unbound orbital electron is always forced by an external magnetic field B_z to move in a plane perpendicular to the z axis.

An inhomogeneous magnetic field B_z, generates a force in the z direction on an unbound orbital electron.

$$\vec{F}_z = (\vec{m} \cdot \frac{\delta}{\delta \vec{r}_z})\vec{B}_z \qquad \vec{m} = I\ \vec{A}\ \cos\theta \qquad I = \frac{e\ \omega}{2\pi} \tag{209}$$

This force is measured in the Stern-Gerlach experiment. The standard model associates an angular momentum to the magnetic field of an orbiting electron. As unbound orbital electrons have no angular momentum, a fictitious angular momentum (spin) was postulated.

The energy splitting in a magnetic field is given by

$$\Delta E = g_l\ m_l\ \mu_B\ B_z \tag{210}$$

with g_l the Lande factor, m_l the quantum number projection of angular momentum, μ_B the Bohr magneton, and B_z the magnetic flux density.

Conclusions: The present approach is based on the "E & R " model, where nucleons are composed of electrons and positrons that neither attract nor repel each other when the distance between them tends to zero. A nucleon can polarize, so that an orbital electron can interact during a short time with more than one positron of the

nucleon. In the case of the hydrogen, the orbital electron can be attracted during a short time by two or more positrons of the proton defining the higher energy levels for the orbital electron.

As nucleons are composed of electrons and positrons, also quarks are composed of electrons and positrons. The fractional charges of quarks are simply the relation between the number of electrons or positrons that integrate the quark, to the total number of electrons and positrons that compose the quark. No fractional charges exist.

The electron shells of atoms is the result of the accommodation of the electrons and positrons of the atomic nuclei in the quarks. The combination principle used in spectroscopy becomes with the "E & R " model a physical interpretation.

4.4 Helium atom.

Fig. 2 shows the Helium atom where one orbital electron interacts with n_1 positrons and the other with n_2 positrons of the nucleus.

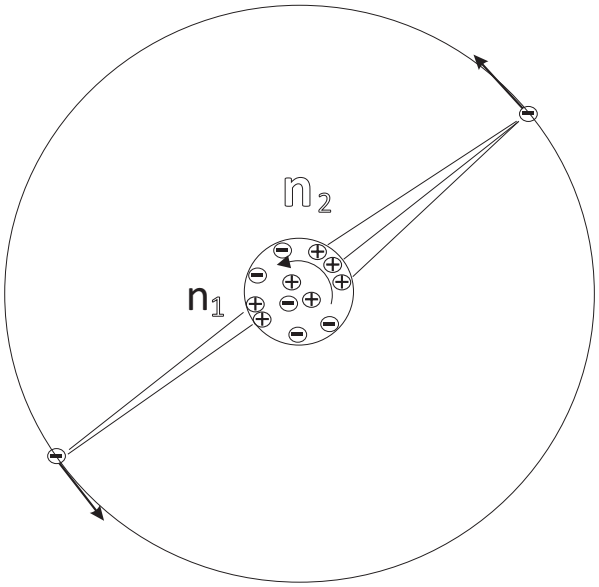

Figure 2: Energy levels at an excited helium atom.

The potential energy of the excited system is given by

$$E_{He} = E_{n_1} + E_{n_2} + E_{1,2} \tag{211}$$

where

$$E_{n_1} = K_u \, \frac{1}{r'_{He}} \, \frac{1}{n^2} \tag{212}$$

$$E_{n_2} = K_u \, \frac{1}{r'_{He}} \, \frac{1}{n^2} \tag{213}$$

$$E_{1,2} = \frac{K_u}{2 \, r'_{He} \, n^2} \tag{214}$$

Note: The present approach explains energy levels with:

- the number n_p of positrons of the nucleus that interact with the orbital electron.
- the quantization of radii of orbital electrons expressed with $r^{'} = r^{'}(n, l, m_l, k)$.

The last explains the energy quantization at the positronium where $n_p = n_e = 1$.

The general explanation is given by the interaction between FPs emitted by external nuclei and orbital electrons, and the own emitted FPs. The quantization of energy levels is finally reduced to the quantization of the energy of each FP.

$$E_{FP} = h \, \nu_{FP} \qquad with \qquad \nu_{FP} \qquad a \;\; universal \;\; constant \tag{215}$$

5 Splitting of atoms and energy levels.

The present approach gives different interpretations for the splitting of atoms at the Stern-Gerlach experiment and the splitting of energy levels at the hydrogen atom.

5.1 Splitting of atoms in the Stern-Gerlach experiment.

To explain the splitting of the atomic ray in the Stern-Gerlach experiment, electrons were assigned an intrinsic spin with a quantized magnetic field that takes two positions, up and down relative to an external magnetic field, although it is not possible to explain how the intrinsic spin and magnetic field are generated. Measurements with individual electrons to detect the magnetic spin are fruitless because of the strong Lorenz force.

Classical physics associates to an orbital electron an angular moment $\vec{l}$ and a magnetic moment $\vec{\mu}$

$$\vec{\mu} = I\vec{A} = -\frac{e}{2m_e}\vec{l} \tag{216}$$

An external field $\vec{B}$ generates a potential magnetic energy E_{pot} and an angular moment $\vec{D}$

$$E_{pot} = -\vec{\mu}\vec{B} \qquad\qquad \vec{D} = \vec{\mu} \times \vec{B} = \frac{d}{dt}\vec{l} \tag{217}$$

If the angular moment $\vec{l} = 0$ we have that $\vec{\mu} = 0$, $E_{pot} = 0$ and $\vec{D} = 0$.

Unbound orbital electrons have in quantum mechanics angular moment $\vec{l} = 0$ what would give an magnetic moment $\vec{\mu} = 0$ and make impossible to explain the splitting of the neutral atom in the Stern-Gerlach experiment. To solve the problem, an intrinsic spin $\vec{s}$ was postulated for the electron with an operator with an eigenstate of the z component of the spin operator with the projection quantum number $m_s = \pm\frac{1}{2}\hbar$ parallel to the external field $\vec{B}$. The magnetic moment then becomes

$$\hat{\vec{\mu_s}} = g_s \, \mu_B \, \frac{\hat{\vec{s}}}{\hbar} \qquad with \qquad \mu_B = -\frac{e\hbar}{2m_e} \tag{218}$$

The postulate of an intrinsic spin makes the magnetic moment $\vec{\mu_s}$ independent of the existence of the angular moment l of the orbital electron and the Stern-Gerlach experiment can be explained.

For the standard model the unbound orbital electron has no angular orbital moment and the generated magnetic field takes the direction of maximum compensation of the external magnetic field. This field is opposed to the external magnetic field what is expressed with the projection quantum number $m_s = \pm 1/2$.

The proposed approach has no unbound orbital electrons because atomic nuclei are composed of electrons and positrons that move with the orbital electron and generate an angular moment $l \neq 0$.

Fig: 3 shows the generation of the magnetic field dH_n independent of the angular moment l of an orbital electron.

The concept is shown in Fig: 3

The approach $E\&R$ UFT shows that electrons and positrons coexist in nucleons without repelling or attracting each other. They can be seen as swarms of electrons and positrons forming the nucleon. As nuclei are composed of nucleons they are also composed of electrons and positrons as shown in Fig. 3 a).

The charge Q of a nucleus is replaced by the expression $\Delta n = n^+ - n^-$ which gives the difference between the **constituent** numbers of electrons and positrons that form the nucleus. As the n_i are integer numbers, the Charge of the nucleus is quantified.

As examples we have for the proton $n^+ = 919$ and $n^- = 918$ with a binding Energy of $E_{B_{prot}} = -6.9489 \cdot 10^{-14}\ J = -0.43371\ MeV$, and for the neutron $n^+ = 919$ and $n^- = 919$ with a binding Energy of $E_{B_{neutr}} = 5.59743 \cdot 10^{-14}\ J = 0.34936\ MeV$.

The dH_n field is generated by the orbital electron and the interacting positron of the nucleus that follows the orbital electron. The two opposed currents generate a dH_n field equal to the field of a bar magnet as shown in Fig. 3 b).

The neutral atoms used in the Stern-Gerlach experiment have all complete shells plus one electron of the next shell, which is not unbound, because it interacts with one positron of the nucleus which follows him. The configuration of the Ag is $[Kr]4d^{10}5s^1$.

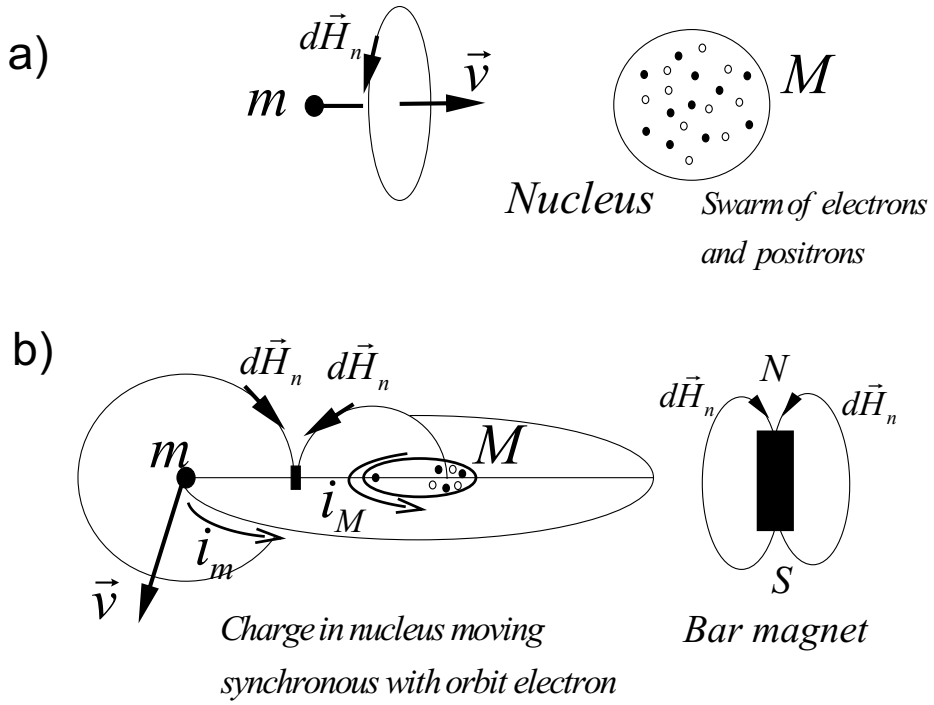

Figure 3: Magnetic field dH_n of an orbital electron.

5.2 The splitting of energy levels at the hydrogen atom.

The proposed approach represents electrons and positrons as focal points of rays of FPs that move from infinite to infinite with light speed and infinite speed. FPs are emitted by focal points and at the same time, FPs emitted by other focal points regenerate them. Focal points that emit FPS with light speed are regenerated by FPs with infinite speed and vice versa. At the focal point the speed of the FPs changes.

We start with (190)

$$E = -\frac{K_u}{r' \left[n^2 + \frac{l}{\alpha} + \frac{k\ \pi}{\alpha\ \ln \frac{r}{r_u}} \right]} \tag{219}$$

with

$$K_u = \frac{e^2}{4\pi\ \epsilon_o} \qquad K_u = \alpha\ \hbar\ c \tag{220}$$

The energy E is defined by three quantum numbers, namely n, l and k. The term in the denominator that is associated with the intrinsic spin of the orbital electron, namely

$$\frac{k\ \pi}{\alpha\ \ln \frac{r}{r_u}} \approx \frac{1}{\alpha}\ \frac{1}{2}\ k = \frac{4\ \pi\ \epsilon_o}{e^2}\ \hbar\ c\ \frac{1}{2}\ k \qquad k = 0, \pm 1, \pm 2, \pm 3, \cdots \tag{221}$$

is a function of the product of the charge of the hydrogen nucleus e and the charge of the orbital electron e, and a function of the integration limits r and r_u, what shows, that the above term is not the product of an intrinsic spin of the orbital electron. It is given by the interaction between nucleus and orbital electron, the same as the orbital angular momentum.

6 Radiation of accelerated particles.

Experience shows that all accelerated charged particles emit energy as electromagnetic radiation. The stability of orbital electrons, which are radially accelerated, is explained with the quantization of the energy levels of orbital electrons.

The present approach explains the origin of energy levels of orbital electrons with the number of positrons of the nucleus that interact with the orbital electron. In other words, the linear superposition of potential fields of positrons, leaving open the question of stability of the radially accelerated orbital electrons.

The E&R model represents subatomic particles (SPs) as focal points of rays of Fundamental Particles (FPs) that move from infinity to infinity. FPs have longitudinal and transversal angular momenta where the energy of the SP is stored. FPs are emitted by the focal point and at the same time regenerate the focal point. Regenerating FPs are those FPs that were emitted previously by external subatomic particles. Because of the energy conservation principle, the current of emitted FPs must be equal to the current of regenerating FPs. SPs interact through the cross product of the angular momenta of their FPs.

The regenerating FPs of a SP are activated by their emitted FPs when they arrive to external SPs. There is a time delay between the emitted FP and the arrival of the regenerating FP that was activated by the first. The emitted FP takes with it the information of the location of the focal point from which it was emitted. The information is stored in the direction of the longitudinal angular momenta. This information is transmitted to the regenerating FP when activated, and allows that the regenerating FP meets the focal point.

At SPs that are at rest or move with constant speed, the externally activated regenerating FPs meet the focal point. At SPs that are accelerated, the externally activated regenerating FPs fail the focal point, because of the acceleration during the time delay. The regenerating FPs that fail the focal point move then independent from the focal point as radiated photons or neutrinos.

In the case of the orbital electron with its radial acceleration, the regenerating FPs don't fail the electron because of the small radius of the orbit. It is equivalent to a

resting electron for all external SPs where the regenerating FPs are activated. Because of the small energy of the orbital electron the uncertainty principle between energy and space includes the orbit of the electron.

$$(\Delta E) \cdot (\Delta x) \geq \frac{1}{2}\, \hbar\, c \tag{222}$$

Example: The energy of the orbital electron of the hydrogen atom with $l = 0$ is $E_e = 3.4250 \cdot 10^{-18}\ J$ which gives an uncertainty of $\Delta x = 4.6182 \cdot 10^{-9}\ m$ which is grater than the diameter of the atom with approximately $2\ a_o = 1.0584 \cdot 10^{-10}\ m$.

7 Stable and unstable particles.

Particles in the SM are classified as Gauge Bosons, Leptons, Quarks, Baryons and Mesons. The classification makes no difference between stable and unstable particles. Unstable particles with energies much grater than the energies of the stable electron ($0.511\ MeV/c^2$), positron or neutrino are defined as Basic Subatomic Particles (BSPs), violating the concept of basic particles which must be the constituents of all not basic particles. The result is the search for basic particles like the unstable Quarks with energies above $0.35\ GeV/c^2$.

The approach "Emission and Regeneration" UFT

1. defines as BSPs the electron, positron and the neutrino which are stable particles, and defines all particles with higher energies, stable or unstable, as Composed Subatomic Particles (CSPs) which are integrated by BSPs.

2. defines electrons and positrons as focal points of rays of Fundamental Particles (FPs) which go from infinite to infinite and have longitudinal and transversal angular momenta. Interactions between electrons and positrons are the result of the interactions of the angular momenta of their FPs. No carrier bosons are required to describe interactions between subatomic particles.

3. defines neutrinos as pairs of FPs with opposed angular momenta which generate linear momenta, and photons as a sequence of pairs of FPs with opposed angular momenta that generate a sequence of opposed linear momenta.

4. shows that no strong forces are required to hold electrons and positrons together, which are the constituents of protons and neutrons. The forces between the constituents electrons and positrons tend to zero for the distance between them tending to zero.

5. shows that weak forces which are responsible for the decay of atomic nuclei are electromagnetic forces.

6. shows that gravitation forces are also electromagnetic forces.

The conclusion is, that all interactions between subatomic particles are electromagnetic interactions and described by QED. Interactions as described by QCD are simply the product of the primitive definition of particles as point-like entities which require carriers to explain their interactions.

7.1 The potentials of the four interactions.

Our SM differentiates between the following potentials to explain interactions between particles.

- Strong
- Weak
- Gravitation
- Electromagnetic

In [11] the momentum curve between two static charged BSAs (electron/positron) was derived resulting Fig. 4 and the following regions were defined:

1. From $0 \ll \gamma \ll 0.1$ where $p_{stat} = 0$
2. From $0.1 \ll \gamma \ll 1.8$ where $p_{stat} \propto d^{\,2}$
3. From $1.8 \ll \gamma \ll 2.1$ where $p_{stat} \approx constant$
4. From $2.1 \ll \gamma \ll 518$ where $p_{stat} \propto \frac{1}{d}$
5. From $518 \ll \gamma \ll \infty$ where $p_{stat} \propto \frac{1}{d^{\,2}}$ (Coulomb)

The static momentum curve of Fig. 4 is part of the potential well of an atomic nucleus as shown in Fig. 5, which can be approximated by a piecewise constant potential for the analytical analysis in quantum mechanics.

The force on electrons or positrons that move in the defined regions of the potential well is given by the following equations derived in [11]:

$$d\bar{F}_{i_n} = \frac{1}{8\,\pi}\,\sqrt{m_p}\,r_{o_p}\,rot\,\frac{d}{dt}\int_{r_r}^{\infty} d\bar{H}_n \qquad with \tag{223}$$

$$\frac{d}{dt}\int_{r_r}^{\infty} d\bar{H}_n = \frac{1}{2}\,\frac{d}{dt}[H_n]\,\frac{r_o}{r_r}\,\sin\varphi\,d\varphi\,\bar{s}_\gamma \;-\; H_n\,v\,\frac{r_o}{r_r^2}\,\sin\varphi\,\cos\varphi\,d\varphi\,\bar{s}_\gamma \tag{224}$$

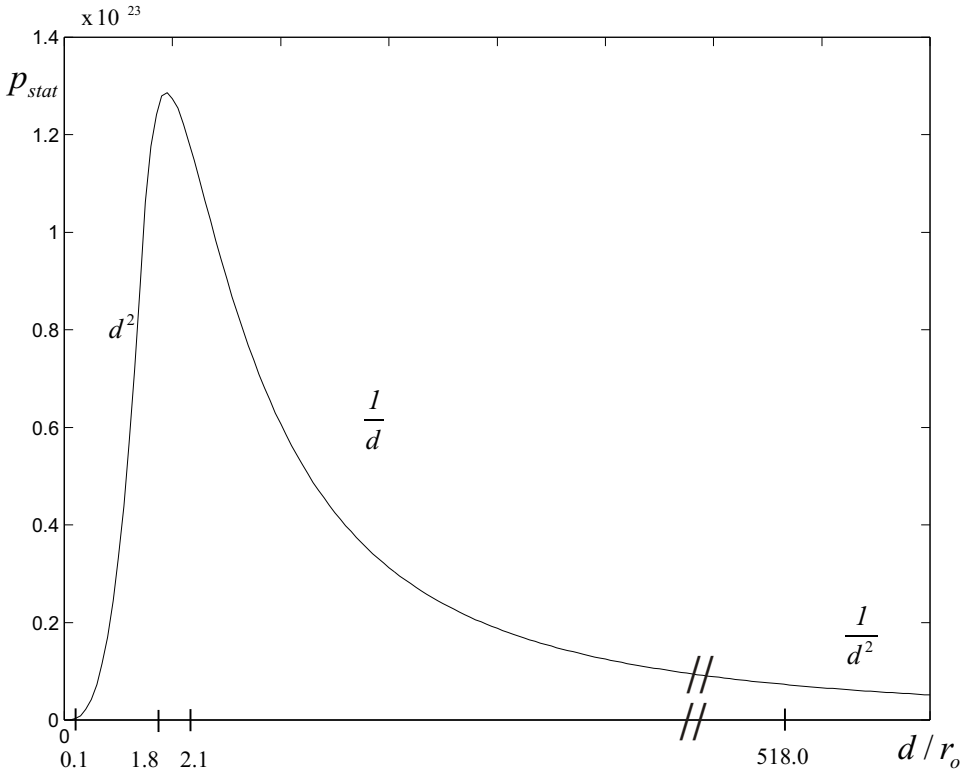

Figure 4: Linear momentum p_{stat} as function of $\gamma = d/r_o$ between two static BSPs with equal radii $r_{o_1} = r_{o_2}$

$$+ \frac{1}{2} H_n \frac{1}{r_r} \sin\varphi \, d\varphi \frac{dr_o}{dt} \bar{s}_\gamma$$

For the regions we have that:

- BSPs that are in region **1** don't attract nor repel each other. The static force is zero and no binding Gluons nor **strong forces** to hold them together are needed.
- BSPs that have migrated slowly from region **1** to region **2** where the potential groves approximately with d^2, are accelerated to or away from the potential wall by the static force according the charge of the particle and the charge of the remaining particles in region **1**. We can differentiate between:
  - BSPs that are accelerated away from the potential wall (region **3**) induce on BSPs of other atoms the **gravitation force**. The accelerated BSPs transmit their acquired momentum to BSPs of other atoms (induction) and stop their movement immediately according the conservation law of momentum. The force on accelerated BSPs is given with $\frac{d}{dt}[H_n] = \sqrt{m}\frac{dv}{dt}$.
  - BSPs that are accelerated to the potential wall may tunnel the wall what results in the decay of the atom with the corresponding radiations. No special **weak force** is required.

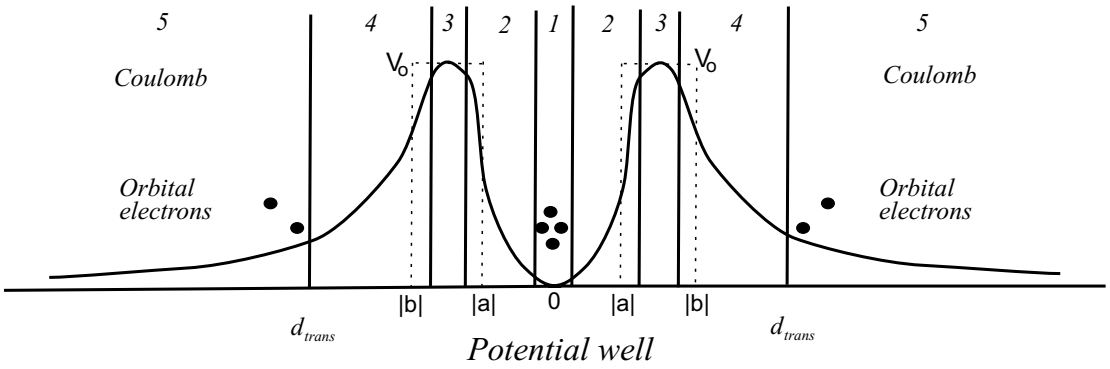

Figure 5: Potential well of an atom.

- BSPs in the region **5** where the Coulomb force exists, orbit around the atom nucleus. This is called in the SM the **electromagnetic force**.

The "Emission & Regeneration" UFT approach shows that all forces are derived from one Field, the dH field. It also shows that all interactions are of electromagnetic type and described by QEDs (Quantum Electrodynamics) and that no other type of interactions are required. It shows that all particles are composed of electrons, positrons and neutrinos and that particles of very short lifetime are composed particles.

8 Compatibility of gravitation with Quantum mechanics.

The potential in which an orbital electron in an Hidrogen atom with $Z = 1$ moves is

$$U(r)_{Coul} = -\left(\frac{Z\,e^2}{4\pi\epsilon_o}\right)\,\frac{1}{r} = 2.3072\cdot 10^{-28}\,\frac{1}{r}\,J \qquad with \quad Z=1 \tag{225}$$

We know from [5] page 178 that the discrete energy levels for the orbital electron of the H-atom is

$$E_{n_{Coul}} = -\,\frac{m}{2\hbar^2}\,\left(\frac{Z\,e^2}{4\pi\epsilon_o}\right)^2\,\frac{1}{n^2} = 2.1819\cdot 10^{-18}\,\frac{1}{n^2}\,J \tag{226}$$

The difference between the energy levels is

$$\Delta E_{n_{Coul}} = 2.1819\cdot 10^{-18}\,\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]\,J \tag{227}$$

8.1 Quantized gravitation.

In the present approach of "Emission & Regeneration" UFT gravitation is presented based on the reintegration of migrated electrons and positrons to their nuclei. According to that model the force on one electron/positron of a mass M_1 due to the reintegration of an electron/positron to an atomic nucleus of a mass M_2 is given by

$$F_i = \frac{dp}{\Delta t} = \frac{k\,c\,\sqrt{m}\,\sqrt{m_p}}{4\,K\,d^{\,2}}\,\int\!\!\int_{Induction} \qquad with \qquad \int\!\!\int_{Induction} = 2.4662 \tag{228}$$

and the corresponding potential is

$$U(r)_{Grav} = \left(2.4662\,\frac{k\,c\,\sqrt{m}\,\sqrt{m_p}}{4\,K}\right)\,\frac{1}{r} = 2.3071\cdot 10^{-28}\,\frac{1}{r}\,J \tag{229}$$

If we write the Schroedinger equation with the gravitation potential instead of the Coulomb potential for the H-atom, we get discrete energy levels simply in replacing the expression in brackets of eq.(226) with the expression in brackets of eq. (229)

$$E_{n_{Grav}} = -\,\frac{m}{2\hbar^2}\,\left(2.4662\,\frac{k\,c\,\sqrt{m}\,\sqrt{m_p}}{4\,K}\right)\,\frac{1}{n^2} = 2.1816\cdot 10^{-18}\,\frac{1}{n^2}\,J \tag{230}$$

In the same model of gravitation the number of reintegrating electrons/positrons for a mass M is derived as $\Delta G = \gamma_G\,M$ with $\gamma_G = 5.3779\cdot 10^8\;kg^{-1}$. The resulting energy level due to all reintegrating electrons/positrons of M_1 and M_2 is

$$E_{n_{Grav\ tot}} = 2.1816 \cdot 10^{-18}\ \Delta G_1\ \Delta G_2\ \frac{1}{n^2}\ J \tag{231}$$

For the H-Atom M_2 is formed by one proton composed of 918 electrons and 919 positrons and M_1 is the mass of the electron. The mass of a proton is $M_2 = m_{prot} = 1.6726 \cdot 10^{-27}\ kg$ and the mass of the electron $M_1 = m_{elec} = 9.1094 \cdot 10^{-31}\ kg$. We get $\Delta G_2 = 8.9951 \cdot 10^{-19}$ and $\Delta G_1 = 4.8989 \cdot 10^{-22}$. We get for the energy difference for orbital electrons at the H-Atom due to gravitation potential

$$\Delta E_{n_{Proton}} = 9.6134 \cdot 10^{-58}\ \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]\ J \tag{232}$$

If we compare the factors of the brackets for the energy difference due to the Coulomb potential of eq. (227) and the gravitational potential of eq. (232), we see that even between very different energy levels n_1 and n_2 of the gravitational levels the energy differences of the gravitation are neglectible compared with the Coulomb.

For the energy difference between two levels n_1 and n_2 of an atom we can write:

$$\Delta E_{n_{Coul}} \pm \Delta E_{n_{Grav}} = h(\nu \pm \Delta\nu) = 2.1819 \cdot 10^{-18}\ [1 \pm \Delta G_1\ \Delta G_2]\ \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]\ J \tag{233}$$

with $\Delta G = \gamma_G\ M$ where $\gamma_G = 5.3779 \cdot 10^8\ kg^{-1}$.

Now we make the same calculations for the difference between the energy levels due to the gravitation potential of the sun with $M_2 = M_\odot = 1.9891 \cdot 10^{30}\ kg$ and the earth with $M_1 = M_\dagger = 5.9736 \cdot 10^{24}\ kg$. We we get $\Delta_{G_\odot} = 1.0697 \cdot 10^{39}$ and $\Delta_{G_\dagger} = 3.2125 \cdot 10^{33}$ resulting

$$\Delta E_{n_{\odot,\dagger}} = 7.4968 \cdot 10^{54}\ \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]\ J \tag{234}$$

As the earth shows no quantization in its orbit around the sun, two adjacent levels n_1 and n_2 must be very large outer levels so that $\Delta E_{n_{\odot,\dagger}} \approx 0$, similar to the large outer levels of the conducting electrons of conducting materials. Mathematically we can write with $n_2 = n_1 + 1$

$$\lim_{n_1 \Rightarrow \infty} \Delta E_{n_{\odot,\dagger}} = 7.4968 \cdot 10^{54}\ \left[\frac{1}{n_1^2} - \frac{1}{(n_1+1)^2}\right] = 0\ \ J \tag{235}$$

8.2 Relation between energy levels and space.

The compatibility of gravitation as the reintegration of migrated electrons/positrons to their nuclei is also shown by the following calculations. From eq. (231) we get the energy difference between two gravitation levels

$$\Delta E_{n_{Grav}} = 2.1816 \cdot 10^{-18}\ \Delta G_1\ \Delta G_2\ \left[\frac{1}{n_1^2}\ -\ \frac{1}{n_2^2}\right]\ \ J \tag{236}$$

and with the difference between two gravitation potentials at different distances

$$\Delta U_{Grav} = G\ M_1\ M_2\ \left[\frac{1}{r_1}\ -\ \frac{1}{r_2}\right]\ \ J \tag{237}$$

we can write that $\Delta E_{n_{Grav}} = \Delta U_{Grav}$ what gives with $r_1\ r_2 \approx r^2$

$$\frac{\Delta r}{r^2} = \frac{2.1816 \cdot 10^{-18}\ \gamma_G^2}{G}\ \left[\frac{1}{n_1^2}\ -\ \frac{1}{n_2^2}\right] \tag{238}$$

For the H-atom with $r \approx 10^{-13}\ m$ we get for the difference between the two first energy levels $n_1 = 1$ and $n_2 = 2$

$$\Delta r = \frac{2.1816 \cdot 10^{-18}\ \gamma_G^2}{G}\ r^2\ \left[\frac{3}{4}\right] = 7.0926 \cdot 10^{-17}\ m \tag{239}$$

what is a reasonable result because $\Delta r << r$.

Now we make the same calculations for the earth and the sun with $r_{\odot,\dagger} \approx 150.00 \cdot 10^9\ m$. We get

$$\Delta r_{\odot,\dagger} = 2.1164 \cdot 10^{32}\ \left[\frac{1}{n_1^2}\ -\ \frac{1}{n_2^2}\right] \tag{240}$$

As the earth shows no quantization in its orbit around the sun, two adjacent levels n_1 and n_2 must be very large outer levels so that $\Delta r_{\odot,\dagger} \approx 0$, similar to the large outer levels of the conducting electrons of conducting materials.

8.3 Superposition of gravitation and Coulomb forces.

The "Emission & Regeneration" UFT shows that the Coulomb and the Ampere forces tend to zero for the distance between electrons/positrons tending to zero. The behaviour is explained with the cross product of the angular momenta of the regenerating rays of FPs that tends to zero.

The induction force is not a function of the cross product but simply the product between angular momenta of the regenerating rays of FPs. The result is that the induction force does not tend to zero with the distance between inducing particles tending to zero. As the gravitation was defined as the reintegration of migrated electrons/positrons to their nuclei and as a induction force, the gravitation force prevails over the Coulomb or Ampere forces for the distance tending to zero.

Fig. 6 shows qualitatively the resulting momentum due to Coulomb/Ampere and Gravitation momenta between an atomic nucleus of a target and a He nucleus.

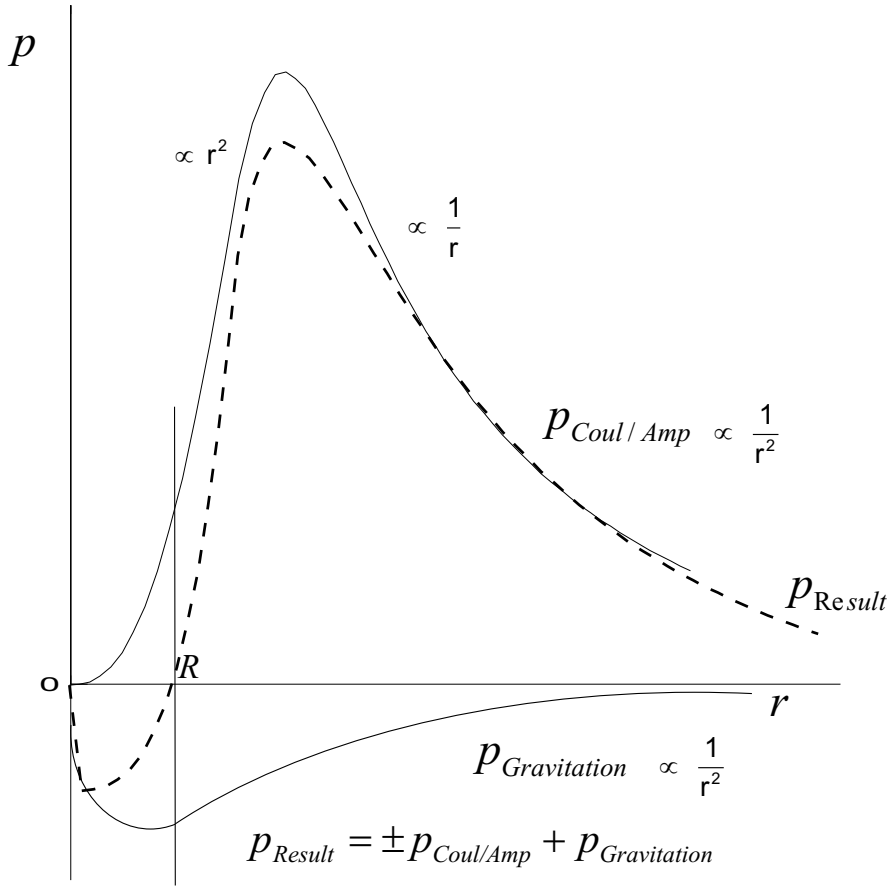

Figure 6: Resulting linear momentum p due to Coulomb/Ampere and Gravitation momenta.

Note: The gravitation model of "Emission & Regeneration" UFT is based on a physical approach of reintegration of migrated electrons/positrons to their nuclei and compatible with quantum mechanics, while General Relativity, the gravitation model of the SM, based on a mathematical-geometric approach is not compatible with quantum mechanics.

9 Table comparing the SM and the 'E & R' model.

Model	Sub-division	Particle representation	Force Carriers	Fields	Interactions	Gauges	Comment
SM (Poly-particle)	Classic	Point-like	Gluons W-Boson Photons Gravitons	Strong Weak Electromagnetic Gravitation	Strong Weak Electromagnetic Gravitation		Four fields, one for each type of force
	QM	Wave Packet				QCD Electroweak QED Gravity Duality	
E & R (Mono-particle)	Classic	***Focal-point*** of rays of Fundamental Particles	***Fundamental Particle*** with Longitudinal and Transversal angular momenta	***dH*** field with Longitudinal and Transversal components	Electromagnetic (Long x Long, Trans x Trans, Trans - Long)		One field for all forces
	QM	Wave Packet				QED	

Figure 7: Table comparing the SM and the 'E & R' model.

Fig. 7 shows the SM and the 'E & R' model subdivided in classical physics and QM. The classic part of the SM with its point-like representation of particles has four force-carriers, four fields and four interactions. QM based on the classical physics of the SM has correspondingly four gauge theories.

The classic part of the 'E & R' model with its focal-point representation of particles has only one type of force-carrier, only one field and only one type of interaction. QM based on the classical physics of the 'E & R' model has correspondingly only one type of gauge theory, namely QED.

The SM has four fields one for each type of force while the 'E & R' model has only one field for all forces and is therfore a UFT.

The SM is a poly-particle model while the 'E & R' model is a mono-particle model.

10 Spin, magnetic moment and photon.

10.1 The spin.

According the E&R model, electrons and positrons are composed of Fundamental Particles (FPs) which have an energy defined by

$$E_{FP} = h\nu_o \tag{241}$$

with ν_o a universal frequency.

The energy of an electron or positron can thus be expressed as

$$E_e = N_e\ E_{FP} \qquad\qquad N_e = \frac{E_e}{E_{FP}} = \frac{\sqrt{E_o^2 + E_p^2}}{E_{FP}} = \frac{E_s + E_n}{E_{FP}} \tag{242}$$

where N_e is the number of FPs that composes the electron or positron. For the non relativistic case we have

$$N_e = \frac{E_e}{E_{FP}} = \frac{E_s + E_n}{E_{FP}} = \frac{1}{E_{FP}}\left[E_o + pc\right] \tag{243}$$

An orbital electron interacts with the nucleus and has an orbital moment given by

$$L = m_e\ \rho\ v_t = \rho\ v_t\ N_e\ \frac{E_{FP}}{c^2} \qquad\qquad m_e = \frac{E_e}{c^2} = N_e \frac{E_{FP}}{c^2} \tag{244}$$

where ρ is the radius of the orbit and v_t the tangential speed.

As the nucleus of the atom is also composed of electrons and positrons which are composed of FPs, the orbital electron can pass or receive FPs from the nucleus. The number of FPs of the orbital electron can thus vary between

$$N = N_e\ \pm\ \Delta N_e \qquad with \qquad \Delta N_e = 0,\ 1,\ 2,\ 3, \cdots \tag{245}$$

We get for the total angular moment of an orbital electron for the case of $N = N_e\ \pm\ 1$

$$\vec{J} = m_e\ \vec{\rho} \times \vec{v}_t = \frac{\nu_o}{c^2}\left[N_e\ h\ \pm\ h\right]\ \vec{\rho} \times \vec{v}_t = \vec{L}\ \pm\ \vec{S} \tag{246}$$

where L is the orbital angular moment and S is the spin of the electron.

The quantum number $\Delta N_e = 0,\ 1,\ 2,\ 3, \cdots$ gives the number of FPs at which the orbital electron is increased or decreased.

Equation (246) includes the relativistic mass increase due to the definition of the mass as

$$m_e = \frac{E_e}{c^2} = N_e\ \frac{E_{FP}}{c^2} = N_e\ \frac{h\ \nu_o}{c^2} \tag{247}$$

Note: According to the shell structure of the Ag atom the individual electron carries no angular momentum and $\vec{L} = 0$. That is because there is no moment of inertia and that the area vector of the orbit aligns immediately parallel to the external magnetic field. According to the E&R model the energy variation at the electrons is due to the variation of the number of FPs given by $\pm\Delta N_e$. For the special case of $\vec{L} = 0$ only a variation $-\Delta N_e$ is possible. The variation of the number of FPs produces a variation of the mass of the electron and consequently a variation of the kinetic energy.

The splitting of the energy level is the product of the interactions between subatomic particles. There is no need to introduce the postulate of S.Goudsmit and G.E.Uhlenbeck.

10.2 The magnetic moment.

The energy of FPs are stored in the angular momentum $\vec{h}$ what generates a magnetic momentum in an external magnetic field.

The charge q_{FP} and the mass m_{FP} of a FP is given with

$$q_{FP} = \frac{e}{N_e} = e\,\frac{E_{FP}}{E_e} \qquad\qquad m_{FP} = \frac{m_e}{N_e} = m_e\,\frac{E_{FP}}{E_e} \tag{248}$$

The magnetic moment of a FP is defined as

$$\vec{\mu}_{FP} = -\,\frac{q_{FP}}{2\,m_{FP}}\vec{h} = -\,\frac{N_e\,q_{FP}}{2\,N_e\,m_{FP}}\vec{h} = -\,\frac{e}{2\,m_e}\vec{h} = \vec{\mu}_B \tag{249}$$

where $\vec{\mu}_B$ is the Bohr magneton.

The potential magnetic energy is defined as

$$H_{mag} = -\,\vec{\mu}\,\vec{B} \tag{250}$$

with

$$\vec{\mu} = -\,g_l\,\mu_B\,\frac{\vec{l}}{h} \tag{251}$$

where $\vec{l}$ is the orbital angular moment.

10.3 The photon.

The photon is defined in the E&R model as a sequence of FPs with opposed angular momenta. The energy of a photon expressed as a function of the energy of a FP is

$$E_{ph} = N_{ph}\,E_{FP} \qquad\qquad E_{FP} = h\,\nu_o \tag{252}$$

where N_{ph} is the number of FPs that integrates the photon. With $E_{ph} = h\ \nu$ we get

$$N_{ph} = \frac{\nu}{\nu_o} = \frac{c}{\lambda\ \nu_o} \qquad\qquad \nu\lambda = c \tag{253}$$

If we take the Hyperfine-shift of $\nu = 1.42\ MHz.$ for $n = 1$ between $F = 1$ and $F = 0$ as caused by one FP, so that $\nu_o = 1.42\ MHz.$, we get that the energy of a FP is

$$E_{FP} = h\ \nu_o = 5.88 \cdot 10^{-9}\ eV \qquad with \qquad \nu_o = 1.42\ MHz \tag{254}$$

11 Summery of main characteristics of the proposed model.

The following abbreviations are used:

1. Basic Subatomic Particles (BSPs) are electrons, positrons and neutrinos.
2. Subatomic Particles (SPs)
3. Fundamental Particles (FPs)

The main characteristics of the proposed model are:

- Subatomic particles (SPs) are represented as focal points of rays of Fundamental Particles (FPs) that go from infinite to infinite. FPs store the energy of the SPs as rotation defining longitudinal and transversal angular momenta.
- FPs are emitted at the focal point and regenerate the focal point. Regenerating FPs are the FPs that were emitted by other focal points in space.
- The charge of a SP is defined by the rotation sense of the longitudinal angular momenta of the emitted FPs.
- The interacting particles for all types of interactions (electromagnetic, strong, weak, gravitation) are the FPs with their longitudinal and transversal angular momenta.
- All known forces are derived from one vector field generated by the longitudinal and transversal angular momenta of fundamental particles.
- All the basic laws of physics (Coulomb, Ampere, Lorentz, Maxwell, Gravitation, bending of particles and interference of photons, Bragg, Schroedinger) are mathematically derived from the proposed model, making sure that the approach is in accordance with experimental data.

- Electrons and positrons neither attract nor repel each other for the distance between them tending to zero. Nucleons are interpreted as swarms of electrons and positrons.
- The coexistence of protons in the atomic nucleus does not require the definition of a special strong force nor additional mediating particles (gluons).
- Quarks are composed of electrons and positrons and the charge Q is the relation between the difference of positrons and electrons of the quark and the total number of electrons and positrons. Q is the relative charge of the quark.
- The emission of particles from a heavy atomic nucleus does not require the definition of a special weak force nor additional mediating particles.
- Gravitation has its origin in the linear momenta induced by the reintegration of migrated electrons and positrons to their nuclei. No special mediating particles are required (gravitons).
- The gravitation force is composed of an induced Newton component and an Ampere component due to parallel currents of reintegrating electrons and positrons. For galactic distances the induced component can be neglected. A positive Ampere component explains the flattening of galaxies' rotation curve (no dark matter is required) and a negative Ampere component explains the expansion of galaxies (no dark energy is required).
- The inertia of particles is explained with the time delay between the emission and the regeneration of FPs. No special mediating particles are required.
- Permanent magnets are explained with the synchronization along a closed path of reintegrating BSPs to their nuclei.
- The splitting of the atomic beam in the Stern-Gerlach experiment is explained with the magnetic field generated by the parallel currents composed of the orbital electron and the current induced in the atomic nucleus. The magnetic spin is not an intrinsic characteristic of the electron.
- Relativity deduced on speed variables instead of space-time variables gives the same equations as special relativity but without the fictitious concepts of time dilation and length contraction. Also the transversal Doppler effect, which was never experimentally detected, doesn't appears.
- The wave character of the photon is defined as a sequence of FPs with opposed transversal angular momenta which carry potential opposed transversal linear momenta.

- Light that moves trough a gravitation field can only lose energy, what explains the red shift of light from far galaxies (no expansion of the universe is required).
- Diffraction of particles such as the Bragg diffraction of electrons is now the result of the quantized interaction of parallel currents.
- As the model relies on BSPs permitting the transmission of linear momenta at infinite speed via FPs, it is possible to explain that entangled photons show no time delay when they change their state.
- The addition of a wave to a particle (de Broglie) is effectively replaced by a relation between the particles radius and its energy.
- The Schroedinger equation is replaced by an equation where the wave function is derived one time versus space and two times versus time in analogy to Newton's second law.
- The uncertainty relation of quantum mechanics derived with the new wave function form pairs of canonical conjugated variables between "energy and space" and "momentum and time".
- The time independent Schroedinger equation results deriving the new wave function two times versus space, the same as for the established wave function.
- The new quantum mechanics theory, based on wave functions derived from the radius-energy relation, is in accordance with the quantum mechanics based on the correspondence principle.
- All interactions are of electromagnetic type and described by QEDs (Quantum Electrodynamics) and no other type of interactions are required.
- The gravitation of the present approach "Emission & Regeneration" UFT is compatible with quantum mechanics, what is not the case with General Relativity, which is the gravitation model of the SM.
- Finally the hypothesis is made that the apparent CMB radiation is a gravitational effect between the mass of the satellite and the signal evaluating part of the satellite, what would explaining the isotropy of the radiation.

12 Bibliography

Note: The present approach is based on the concept that fundamental particles are constantly emitted by electrons and positrons and constantly regenerate them. As the concept is not found in mainstream theory, no existing paper can be used as reference.

1. Guenter Lehner. **Elektromagnetische Feldtheorie.** 7. bearbeitete Auflage 2010. Springer Verlag.

2. Hering · Martin · Stohrer. **Physik fuer Ingenieure.** Vierte, verbesserte Auflage 1992. VDI Verlag.

3. Albrecht Lindner. **Grundkurs Theoretische Physik.** Teubner Verlag, Stuttgart 1994.

4. Georg Joos. **Lehrbuch der Theoretischen Physik.** 15., voellig neu bearbeitete Auflage 1989. AULA-Verlag Wiesbaden.

5. Max Schubert / Gerhard Weber. **Quantentheorie, Grundlagen und Anwendungen.** Spektrum, Akad. Verlag 1993.

6. Harald Klingbeil. **Electromagnetische Feldtheorie.** 2. ueberarbeitete Auflage 2011, Vieweg + Teubner Verlag.

7. Martin Ammon / Johanna Erdmenger. **Gauge/Gravity Duality.** Cambridge University Press 2015.

8. Benenson · Harris · Stocker · Lutz. **Handbook of Physics.** Springer Verlag 2001.

9. B.R. Martin & G. Shaw. **Particle Physics.** John Wiley & Sons 2003.

10. Stephen G. Lipson. **Optik.** Springer Verlag 1997.

11. Osvaldo Domann. **"Emission & Regeneration" Unified Field Theory.** June 2003. www.odomann.com.